Inventory and Supply Chain Management with Carbon Emissions

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Dedication

To my parents who are always understanding, always supportive,
and most of all always allowing me to pursue my dreams.

Abstract

Although the literature on carbon emissions, from fields such as environmental economics, public policy, and industrial ecology among others, is quite extensive, the literature in supply chain management is relatively limited. Moreover, in addressing concerns about carbon emissions, much of the focus has been on technological fixes (e.g., more carbon-efficient technologies and alternative sources of energy). Much less attention has been paid to the potential of reducing carbon emissions via adjustments in supply chain design and operation. This research, utilizing optimization, game theory, deterministic and stochastic modeling, and mechanism design, aims to bridge this gap. The first part of the study, using the economic order quantity (EOQ) model framework, suggests that it is possible to reduce emissions by modifying order quantities, and provides conditions under which the relative reduction in emissions is greater than the relative increase in cost. The second part examines the extent to which penalizing the emission of harmful pollutants can successfully reduce overall emissions in decentralized supply chains, and shows that requiring each firm to pay for the emissions for which it is directly responsible can paradoxically lead to higher overall supply chain emissions and for this emission to increase in the price of emissions. The third part includes preliminary results including the impact of price variability as well as consumers' preferences for low emission products on the ways firms manage their inventory and the corresponding emissions.

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Chapter 1

Introduction

This thesis consists of two completed papers and some partially completed work. The abridged version of Chapter 2 is published in the journal *Operations Research Letters*. The completed paper in Chapter 3 is currently under review with a refereed journal. The partially completed work, presnted in Chapter 4, forms the basis of the proposed future work. All the work presented here is joint work with my advisor Saif Benjaafar. Chapter 2 is also joint work with Adel Elomri.

A common thread that ties all the work described in this thesis is an attempt to incorporate concern about carbon emissions in the management of supply chains. Although the literature on carbon emissions, from fields such as environmental economics, public policy, and industrial ecology among others, is quite extensive, the literature in supply chain management is relatively limited (see the literature review in Chapter 3). Moreover, in addressing concerns about carbon emissions, much of the focus has been on technological fixes (e.g., more carbon-efficient technologies and alternative sources of energy). Much less attention has been paid to the potential for reducing carbon emissions via adjustments in supply chain design and operation.

In this thesis, we are motivated by two broad sets of questions. The first: "is there an opportunity to make adjustments to the way supply chains are designed and managed that could lead to significant reductions in emissions and without resorting to investments in new technology?" The second: "what is the impact of various environmental regulations, such as carbon taxes, emission caps, and cap-and-trade, on the way supply chains are designed and managed and the corresponding cost/profitability?" In Chapter 2, we address the first question. In Chapters 3 and 4, we address the second.

In particular, in Chapter 2, we provide analytical support for numerical results provided in a recent paper by Benjaafar et al. (2010) suggesting that it is possible, by making only operational adjustments, to significantly reduce carbon emissions without significantly increasing cost. We do so using the framework of the economic order quantity (EOQ) model. We provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and discuss factors that affect the difference in the magnitude of emission reduction and cost increase. We discuss the broader applicability of these results to other operational models and to settings with different carbon pricing schemes.

In Chapter 3, we examine the extent to which pricing emissions of harmful pollutants can successfully reduce overall emissions in decentralized supply chains (supply chains consisting of multiple independent firms or of independent business units within a single firm). In particular, we show that requiring each firm to pay for the emissions for which it is directly responsible can paradoxically lead to higher overall supply chain emissions and for this emission to increase in the price of emissions. We show that this is because firms, in their efforts to reduce their own emissions, can make decisions that adversely impact the emissions of their suppliers or customers. We illustrate how this could arise in the context of a buyer-vendor supply chain where the actions of each firm correspond to operational decisions regarding the frequency and size of replenishment orders, and provide conditions under which higher emission prices can lead to higher total emissions. We then discuss schemes under which this paradox can be eliminated, including those involving coordinating the supply chain, sharing emission responsibility across the supply chain, and penalizing consumption instead of production. We also discuss examples of other settings where the paradox arises, including a setting where a retailer decides on store locations and one where a manufacturer decides on how much to outsource. Finally, we provide a general model for understanding how the actions of one firm could affect the costs and emissions of other parties in the supply chain of which the previous examples can be considered as special cases. We also provide a general necessary and sufficient condition for when higher emission penalties lead to higher supply chain emissions.

In chapter 4, we briefly summarize our preliminary work on two problems. The first is motivated by settings where carbon prices are stochastic, as in a cap-and-trade system. Using the model described in Chapter 2, we examine the impact of carbon price variability. Perhaps surprisingly, we show that higher variability can be beneficial. The second problem, which is also built on the model described in Chapter 2, considers a setting where consumer demand is sensitive to the firm's emissions. We study how the sensitivity of consumers to emissions impact the firm's decisions and profitability.

Each chapter is self-contained, with related literature, notation, and ideas for future research. In most cases, and unless explicitly stated, we maintain the same notation, and the same definition of various concepts throughout the various chapters.

Chapter 2

The Carbon-Constrained EOQ

2.1 Introduction

In pursuing carbon emission reduction efforts, firms have focused for the most part on reducing emissions due to the physical processes involved (e.g., replacing energy inefficient equipment and facilities, redesigning products and packaging, deployment and use of less polluting sources of energy); see [16], the unabridged version of the paper. These efforts are clearly valuable. However, they can overlook a potentially significant source of emissions, one that is driven by business practices and operational policies. In this paper, we examine the extent to which operational adjustments alone could indeed be effective in reducing emissions. We also examine the extent to which such adjustments could take place without significantly increasing cost. This is important because resistance to environmental regulation has often been based on concerns that such regulation would lead to significantly higher costs.

Our analysis is in part motivated by a recent paper [5], in which the authors observe that it is possible to significantly reduce carbon emissions without significantly increasing cost by making only operational adjustments. Their observations are based on numerical results obtained for a lot sizing problem in which a firm decides on production/procurement quantities over a finite planning horizon consisting of discrete periods. In this paper, we use the framework of the economic order quantity (EOQ) model to provide analytical support for similar observations. We provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and describe when the difference between the two is maximized. We discuss the applicability of these results to systems operating under a variety of regulatory policies and to other operational models. We show that, the key requirements are that the cost and emission functions yield different optimal solutions, implying that the cost tradeoffs are different from the emission tradeoffs, and that the cost function is flat around the optimal solution but can be steep elsewhere. We show that significant reductions in emissions can indeed be achieved without significant increases in cost whenever the flat region of the cost function coincides with the steep region of the emission function. In the unabridged version of the paper [16], we show that these features are present in other operational models, including the facility location and newsvendor models, among others.

The results in this paper indicate that the opportunity for reducing carbon emissions via operational adjustments exists whenever the operational drivers of emissions are different from the operational drivers of costs. In settings where this is not the case (e.g., operational decisions that reduce cost tend to also reduce emissions), operational adjustments will obviously be ineffective. In that case, investments in efforts that modify the emission function (e.g., investments in efforts or technologies that lead to reductions in the emission parameters of underlying processes and activities) would be necessary.

Although there is growing literature that is concerned with issues of sustainability in operations (see [5] for a review), papers that explicitly consider emissions are relatively few. Hua et al. [38] consider a model similar to the cap-and-price model we consider in Section 3. They compare the cost and order quantity obtained under cap-and-price to those obtained without carbon considerations. Cachon [9] studies the emission tradeooffs associated with the size and location of retail facilities and shows that carbon pricing would have little impact on these decisions; see also [30] for related analysis.

2.2 Problem Formulation and Results

Consider a firm that faces a constant demand with rate D per unit time. Each time the firm places an order (either with its internal production facility or with an external supplier), it incurs a fixed cost A per order. The firm also incurs a holding cost h per unit kept in inventory per unit time, and a cost c per unit purchased or produced. Without loss of generality, we assume that orders are delivered with zero leadtime (a positive leadtime can be included and does not affect the solution to the problem); we also assume that the firm must satisfy all the demand (the analysis can be easily extended to settings with backorders).

Total cost per unit time is then given by

$$\frac{AD}{Q} + \frac{hQ}{2} + cD.$$

Similar to cost, emissions are associated with ordering, inventory holding, and production/purchasing, with $\hat{A}, \hat{h}$ and $\hat{c}$ denoting the amount of carbon emissions associated per order initiated, per unit held in inventory per unit time, and per unit purchased or produced. Total emission per unit is therefore

$$\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D.$$

The parameterization of emissions under the above formulation is flexible and can be used to capture a variety of settings. For example, if emission from holding inventory depends only on the maximum amount of inventory held (which would correspond to Q), then total emission can be expressed as $\frac{\hat{A}D}{Q} + \hat{h}Q + \hat{c}D$, which is equivalent to the original expression but with an inventory holding emission parameter equal to $2\hat{h}$. Similarly, if emission from holding inventory is invariant to the amount of inventory held, then total emission reduces to $\frac{\hat{A}D}{Q} + \hat{h} + \hat{c}D$, which is equivalent to the original model but with an inventory holding emission parameter equal to 0. If the emission associated with initiating an order has both a fixed and a variable component, say of the form $\hat{A}_1 + \hat{A}_2Q$, then the corresponding total emission is $\frac{\hat{A}_1D}{Q} + \frac{\hat{h}Q}{2} + (\hat{c} + \hat{A}_2)D$, which again has the same form as the original model. Note that, depending on the setting, $\hat{A}$ can be higher or lower than $\hat{h}$. For example, for some products transportation-related emissions are high but storage emissions are low or even negligible (e.g., canned foods) while for others the reverse may be true (e.g., refrigerated foods). Walmart recently discovered that the refrigerants used in grocery stores accounted for a larger percentage of Walmart's greenhouse gas footprint than its truck fleet (`http://www.walmartstores.com/sites/responsibility-report/2012/sustainableFacilities.aspx`). Tesco, the largest retailer in the UK, found that 26 percent of its direct emissions were due to refrigerant leakage while only 12 percent were due to transportation (`http://www.tesco.com/climatechange/carbonFootprint.asp`). Our analysis and results are applicable in all cases (see the end of this section for additional discussion).

The objective of the firm is to choose an order quantity Q that minimizes its cost per unit time subject to the constraint on the amount of carbon emitted (this cap can reflect either government regulations imposed on the firm or a voluntary effort by the firm to reduce its emissions by a specified amount). The amount of carbon emitted is constrained to be less than a certain cap C. The problem can then be formally stated as follows:

$$\text{Minimize } Z(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD \tag{2.1}$$

$$\text{subject to } \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \leq C \tag{2.2}$$

Let $\hat{Q}_{min}$ denote the order quantity that minimizes carbon emission (the *emission-optimal* solution), then it is easy to verify that $\hat{Q}_{min} = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$ and the corresponding emission level is $E_{min} = \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. Consequently, the problem admits a feasible solution if and only if $C \geq E_{min}$. In the remainder, we assume that this condition is always satisfied. Also, let Q^* denote the order quantity that minimizes the total cost while ignoring the carbon emission constraint (the *cost-optimal* solution). Then, it is easy to see that $Q^* = \sqrt{\frac{2AD}{h}}$, which corresponds to the standard EOQ solution. The following theorem characterizes the optimal solution to (2.1)-(2.2).

Theorem 1 *Let*

$$Q_1 = \frac{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \text{ and } Q_2 = \frac{\hat{C} + \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \tag{2.3}$$

where $\hat{C} = C - \hat{c}D$. Then the optimal solution to problem (2.1)-(2.2) is

$$\hat{Q}^* = \begin{cases} Q^* & \text{if } \; Q_1 \le Q^* \le Q_2, \\ Q_1 & \text{if } \; Q^* \le Q_1, \\ Q_2 & \text{if } \; Q^* \ge Q_2. \end{cases} \tag{2.4}$$

Furthermore, the emission level under the optimal order quantity is

$$E(\hat{Q}^*) = \begin{cases} E_{max} & \text{if } Q_1 \le Q^* \le Q_2 \\ C & \text{otherwise,} \end{cases} \tag{2.5}$$

where

$$E_{max} = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D \tag{2.6}$$

and corresponds to the emission level in the absence of the carbon constraint (also corresponds to the emission level when the optimal order quantity is Q^*).

Proof: From constraint (2.2), we can show that the optimal order quantity must satisfy $Q_1 \le \hat{Q}^* \le Q_2$. If $Q_1 \le Q^* \le Q_2$, then obviously $\hat{Q}^* = Q^*$ and the corresponding emission is

$$E(Q^*) = \frac{\hat{A}D}{Q^*} + \frac{\hat{h}Q^*}{2} + \hat{c}D = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D.$$

If $Q^* \le Q_1$ then $\hat{Q}^* = Q_1$ because $Z(Q)$ is convex in Q and choosing a higher value for $\hat{Q}^*$ will lead to a higher cost. Similarly, if $Q^* \ge Q_2$ then $\hat{Q}^* = Q_2$ because choosing a lower value for $\hat{Q}^*$ will lead to higher cost. In both of these cases, constraint (2.2) is binding and, therefore, $E(\hat{Q}^*) = C$. $\square$

In the following proposition, we show that cost is indeed decreasing and convex in the emission cap C while emission is linearly increasing in C, implying that reducing the emission cap leads initially to a larger relative emission reduction than the relative cost increase (e.g., in the example illustrated in Figure 2.1, an emission reduction of 20% leads only to a 4% increase in cost).

Proposition 1 *For $C \geq E_{min}$, emission is linearly increasing in C while cost is decreasing and convex in C.*

Proof: For emission, the result follows immediately from Theorem 1. For cost, consider first the case where $Q^* = \hat{Q}^*$. In this case, cost and emissions are unaffected by the cap C and the results hold. Next, consider the case where $Q_1 \geq Q^*$. In this case, we have

$$\begin{aligned}
Z(\hat{Q}^*) &= Z(Q_1) = \frac{AD\hat{h}}{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}} + \frac{h(\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})}{2\hat{h}} + cD. \\
\frac{\partial Z(\hat{Q}^*)}{\partial C} &= \frac{\partial Z(\hat{Q}^*)}{\partial \hat{C}} = \frac{\partial}{\partial \hat{C}}\left(\frac{AD\hat{h}}{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}} + \frac{h(\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})}{2\hat{h}}\right) \\
&= \frac{A\hat{h} + \hat{A}h}{2\hat{A}\hat{h}} + \frac{A\hat{h} - \hat{A}h}{2\hat{A}\hat{h}} \frac{\hat{C}}{\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}. \\
\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} &= \frac{\partial^2 Z(\hat{Q}^*)}{\partial \hat{C}^2} = \frac{A\hat{h} - \hat{A}h}{2\hat{A}\hat{h}} \frac{-2\hat{A}\hat{h}D}{(\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})^3} = \frac{(\hat{A}h - A\hat{h})D}{(\sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D})^3}.
\end{aligned}$$

Since $Q_1 \geq Q^*$ implies $\sqrt{\frac{2AD}{h}} \leq Q_1 \leq \sqrt{\frac{2\hat{A}D}{\hat{h}}}$, we have $\hat{A}h - A\hat{h} \geq 0$. Therefore, $\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} \geq 0$. In the remaining case of $Q_2 \leq Q^*$ we can show using similar arguments that $\frac{\partial^2 Z(\hat{Q}^*)}{\partial C^2} \geq 0$. Consequently, the optimal cost function is convex with respect to the carbon cap C. $\Box$

From Proposition 1, we can see that imposing an emission cap leads to an emission reduction only if the cap is sufficiently small, namely $C \leq E_{max}$. Otherwise, the

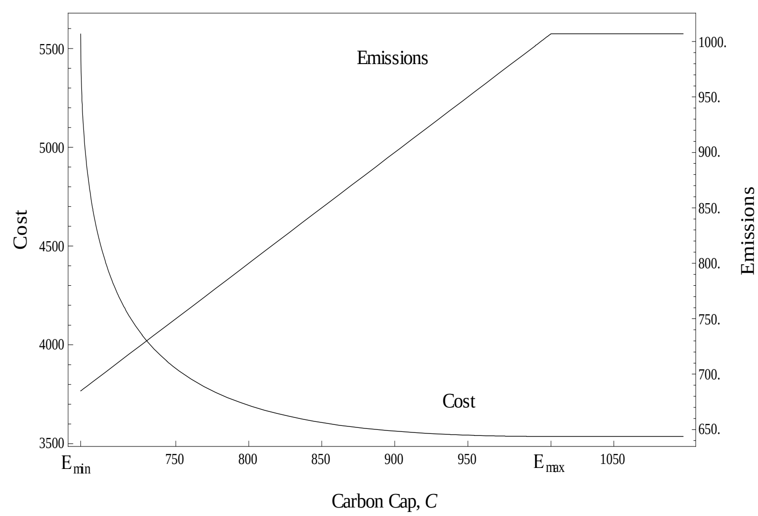
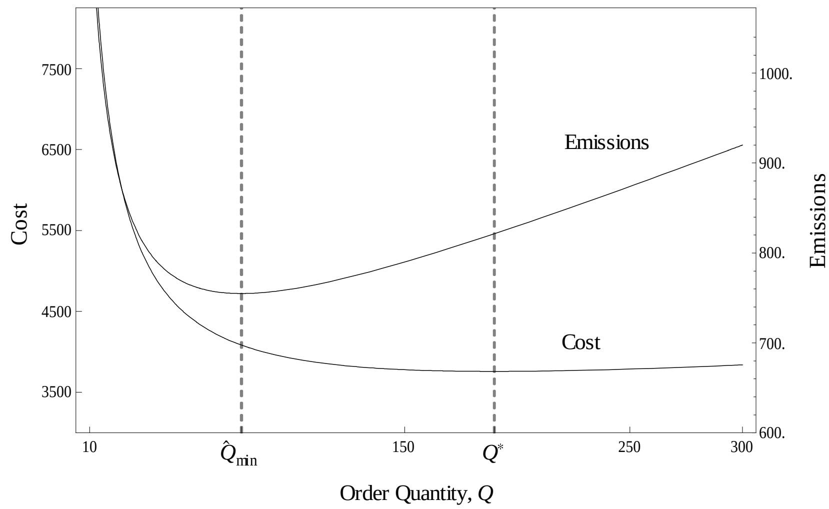


Figure 2.1: The impact of the carbon cap on emission and cost $(D = 600, A = 120, h = 2, c = 5, \hat{A} = 2, \hat{h} = 3, \hat{c} = 1)$

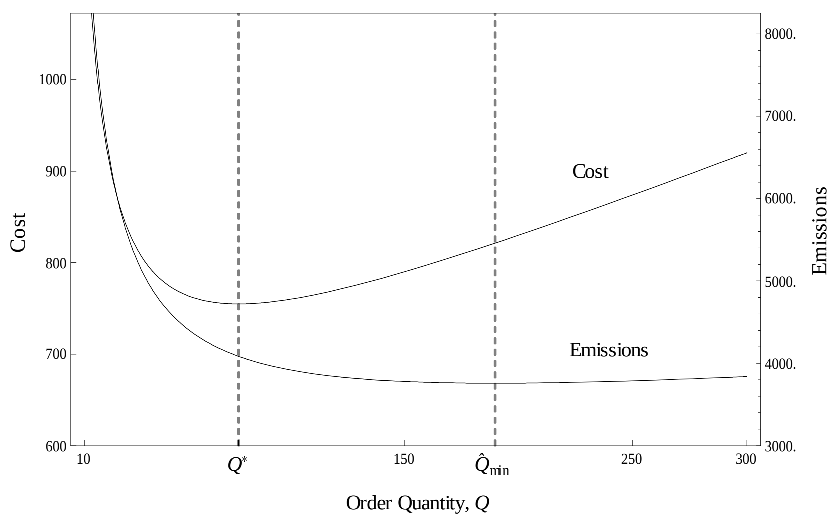
cost-optimal solution is feasible and the corresponding emission is E_{max}. If $C \leq E_{max}$, emission is reduced by adjusting the order quantity (by either increasing it or decreasing it from the cost-optimal order quantity). However, for this to be possible, the cost-optimal order quantity must be different from the emission optimal solution. This leads to the following important corollary.

Corollary 1 *Reducing emissions by adjusting order quantities is possible if and only if* $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$.

This corollary follows from the fact that if $\frac{A}{h} = \frac{\hat{A}}{\hat{h}}$ then the cost-optimal solution is also emission-optimal (i.e., $Q^* = \hat{Q}_{min}$). In that case, emissions are already at their minimum and there is no operational adjustment that could further reduce them. On other hand, if $\frac{A}{h} \neq \frac{\hat{A}}{\hat{h}}$, there is an opportunity to reduce emissions by either increasing or decreasing the order quantity. In particular, if $\frac{A}{h} < \frac{\hat{A}}{\hat{h}} (\frac{A}{h} > \frac{\hat{A}}{\hat{h}})$



(a) $(D = 600, A = 120, h = 4, c = 5, \hat{A} = 10, \hat{h} = 2, \hat{c} = 1)$



(b) $(D = 600, A = 10, h = 2, c = 1, \hat{A} = 120, \hat{h} = 4, \hat{c} = 5)$

Figure 2.2: Cost versus Emissions

then increasing (decreasing) the order quantity decreases emissions; see Figure 3.4 for a graphical illustration.

The broader implication of Corollary 1 is that operational adjustments can lead to emission reductions only if the cost parameters are not strongly correlated to the emission parameters, so that what drives cost more is not what also drives emissions more. This is the case for example when fixed costs are higher than inventory holding costs but fixed emissions are lower than inventory-related emissions.

Although Corollary 1 identifies settings where it is possible to reduce emissions by adjusting order quantities, it does not specify the extent to which this reduction can be realized without significantly increasing cost. The examples shown in Figures 2.1 and 3.4 do suggest that indeed a modest reduction in the order quantity (away from the cost-optimal order quantity) leads to a modest increase in cost but a significant reduction in emission (both cost and emission have similar functional forms, with both being convex in Q and approaching ∞ as Q approaches either 0 or ∞). More importantly, the cost function for the EOQ is *flat* in the region around the optimal solution. This means that in this region a relative change in the order quantity leads to a lower relative increase in cost. In what follows, we characterize this flatness and identify a condition under which the reduction in emission is greater than the increase in cost (for further discussion and related results see [24, 43, 58]).

Let $Z'(Q)$ and $E'(Q)$ refer to the components of cost and emission that are affected by order quantity. That is, $Z'(Q) = AD/Q + hQ/2$ and $E'(Q) = \hat{A}D/Q +$

$\hat{h}Q/2$. Also, let

$$\delta_Q = \frac{Q - Q^*}{Q^*}$$

denote the relative change in the order quantity (with respect to the cost-optimal order quantity), and

$$\delta_Z = \frac{Z'(Q) - Z'(Q^*)}{Z'(Q^*)} \text{ and } \delta_E = \frac{E'(Q^*) - E'(Q)}{E'(Q^*)}$$

denote respectively the corresponding relative change in cost and emission. Then, we can show that

$$\delta_Z = \frac{\delta_Q^2}{2(1+\delta_Q)} \text{ and } \delta_E = -\frac{(1-\alpha)\delta_Q + \delta_Q^2}{(1+\alpha)(1+\delta_Q)}, \tag{2.7}$$

where $\alpha = \frac{\hat{A}/\hat{h}}{A/h}$ is the ratio of the emission parameters to the cost parameters.

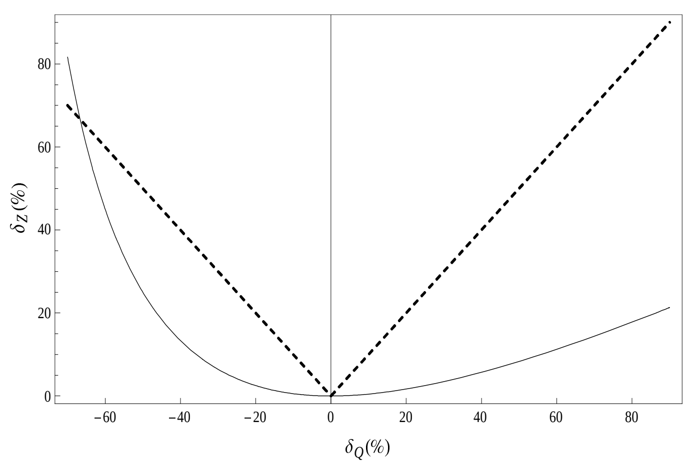


Figure 2.3: The impact of changes in order quantity on cost (Values of δ_Z below the 45 degree lines (shown in dashes) correspond to cases where δ_Z is smaller than δ_Q)

First, it is easy to see that even relatively large values of $|\delta_Q|$ lead to relatively small values of δ_Z. For example, increasing the order quantity by 30% ($\delta_Q = 0.3$)

leads to an increase in cost of only 3.46% ($\delta_Z = 0.0346$); see Figure 2.3 for a full characterization of δ_Z as a function of δ_Q. Mathematically, we can show that

$$\frac{\partial \delta_Z}{\partial \delta_Q} = \frac{\delta_Q(2+\delta_Q)}{2(\delta_Q+1)^2},$$

which is equal to 0 for $\delta_Q = 0$. Moreover, we can show that if $\delta_Q > 0$, then $\delta_Z \leq \delta_Q$ (in other words, the relative increase in cost is always lower than the relative increase in the order quantity regardless of the size of the increase). If $\delta_Q < 0$, then $\delta_Z \leq |\delta_Q|$ as long as $\delta_Q \geq -2/3$ (i.e., we can reduce order quantity by as much as 2/3 without increasing cost by as much). In contrast,

$$\frac{\partial \delta_E}{\partial \delta_Q} = \frac{\alpha - (1+\delta_Q)^2}{(1+\alpha)(1+\delta_Q)^2}$$

and takes on a value equal to $\frac{\alpha-1}{\alpha+1} \neq 0$ for $\alpha \neq 1$ when $\delta_Q = 0$. That is, while the cost function is always flat around Q^*, the emission function can be quite steep (i.e., $|\frac{\alpha-1}{\alpha+1}|$ can be large). The fact that the flat region of the cost function coincides with the steep region of the emission function means that there is an opportunity to achieve more relative emission reductions than the corresponding relative increase in cost.

Next, we describe, the range over which order quantity can be adjusted while guaranteeing that the relative increase in cost is less than the relative reduction in emission, that is $\delta_Z \leq \delta_E$ and $\delta_E \geq 0$.

Proposition 2 *For $\alpha > 1$, $\delta_Z \leq \delta_E$ if $0 \leq \delta_Q \leq \frac{2(\alpha-1)}{3+\alpha}$, and $\delta_Z \geq \delta_E$ otherwise. For $\alpha < 1$, $\delta_Z \leq \delta_E$ if $\frac{2(\alpha-1)}{3+\alpha} \leq \delta_Q \leq 0$, and $\delta_Z \geq \delta_E$ otherwise.*

The proof immediately follows from the expressions of δ_Z and δ_E in Equation (2.7) and for brevity, we omit the details. As we can see, the interval over which the order quantity can be varied depends solely on α with its width increasing in the absolute value of the difference between $\frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h}$. In the limit cases of either $\alpha \to 0$ or $\alpha \to \infty$ the order quantity can be adjusted by as much as a factor of 3. When $\delta_Z = \delta_E$ (and $\delta_Q = \frac{2(\alpha-1)}{3+\alpha}$) the resulting relative decrease in emission and in cost is given by

$$\delta_{E=Z} = \frac{2(\alpha-1)^2}{(1+3\alpha)(3+\alpha)} \tag{2.8}$$

which is also increasing in the absolute value of the difference between the ratios $\frac{\hat{A}}{\hat{h}}$ and $\frac{A}{h}$. In the limit, as either $\alpha \to 0$ or $\alpha \to \infty$, $\delta_{E=Z} \to 2/3$, implying that emissions could be reduced by up to 2/3 without increasing cost by as much.

The following proposition further characterizes the tradeoff between cost and emission reductions.

Proposition 3 *Let $\delta_E(\delta_Z, \alpha)$ denote the relative emission reduction as a function of the relative cost increase δ_Z and α. Then,*

$$\delta_E(\delta_Z,\alpha) = \begin{cases} -\frac{(\delta_Z+\sqrt{2\delta_Z+\delta_Z^2})(1-\alpha+\delta_Z+\sqrt{2\delta_Z+\delta_Z^2})}{(1+\alpha)(1+\delta_Z+\sqrt{2\delta_Z+\delta_Z^2})}, & \text{if} \quad \alpha > 1, \\ \\ -\frac{(\delta_Z-\sqrt{2\delta_Z+\delta_Z^2})(1-\alpha+\delta_Z-\sqrt{2\delta_Z+\delta_Z^2})}{(1+\alpha)(1+\delta_Z-\sqrt{2\delta_Z+\delta_Z^2})}, & \text{if} \quad \alpha < 1. \end{cases} \tag{2.9}$$

Moreover,

- *$\delta_E(\delta_Z, \alpha)$ is concave in δ_Z, and it achieves its maximum value (the emission optimal solution) for $\delta_Z = \frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}$ leading to an emission reduction of $\delta_E = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$,*

- *for $\alpha > 1$, $\delta_E(\delta_Z, \alpha)$ is increasing concave in α (the same percentage increase in cost leads to a lower decrease on emission for a greater value of α), and*

- *for $\alpha < 1$, $\delta_E(\delta_Z, \alpha)$ is decreasing convex in α (the same percentage increase in cost leads to a lower decrease on emission for a lower value of α).*

Proof: Expressing δ_Q as function of δ_Z leads to:

$$\delta_Q = \begin{cases} \delta_Z + \sqrt{2\delta_Z + \delta_Z^2} \geq 0, \\ \\ \delta_Z - \sqrt{2\delta_Z + \delta_Z^2} \leq 0. \end{cases}$$

Substituting into the expression of δ_E leads to (2.9). It is easy to verify that

$$\frac{\partial \delta_E(\delta_Z,\alpha)}{\partial \alpha} = \begin{cases} \frac{2\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^2} \geq 0, & \alpha > 1, \\ \\ -\frac{2\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^2} \leq 0, & \alpha < 1. \end{cases}$$

and

$$\frac{\partial^2 \delta_E(\delta_Z,\alpha)}{\partial \alpha^2} = \begin{cases} -\frac{4\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^3} \leq 0, & \alpha > 1, \\ \\ \frac{4\sqrt{\delta_Z(\delta_Z+2)}}{(1+\alpha)^3} \geq 0, & \alpha < 1. \end{cases}$$

Similarly,

$$\frac{\partial^2 \delta_E(\delta_Z, \alpha)}{\partial \delta_Z^2} = \begin{cases} \frac{(1-\alpha)}{(1+\alpha)(\delta_Z(\delta_Z+2))^{3/2}} \leq 0, & \alpha > 1, \\ \\ \frac{(\alpha-1)}{(1+\alpha)(\delta_Z(\delta_Z+2))^{3/2}} \leq 0, & \alpha < 1. \end{cases}$$

Furthermore, solving for $\frac{\partial \delta_E(\delta_Z,\alpha)}{\partial \delta_Z} = 0$ leads to $\delta_Z = \frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}$ and $\delta_E(\frac{(1-\sqrt{\alpha})^2}{2\sqrt{\alpha}}, \alpha) = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$, which completes the proof. $\square$

The cost-emission tradeoff is illustrated in Figure 2.4, where values of δ_E above the 45 degree lines (shown in dashes) correspond to cases where δ_E is larger than δ_Z. Figure 2.4 suggests that there is a value of δ_Z (and corresponding order

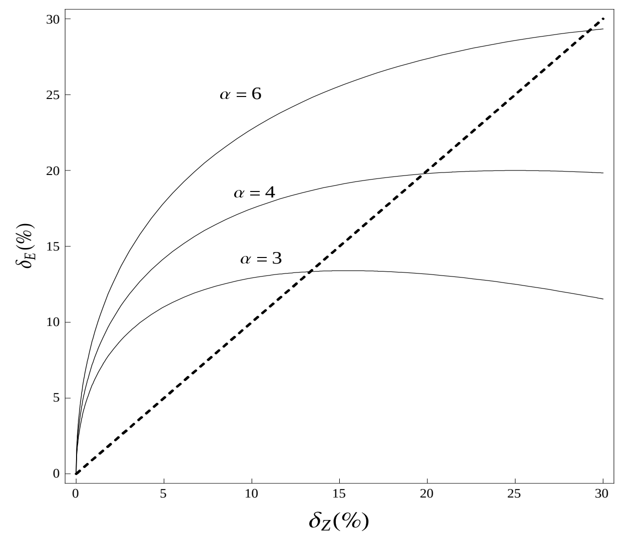


Figure 2.4: Cost-emission tradeoff

quantity) which maximizes the difference $\delta_E - \delta_Z$. This value of δ_Z, to which we refers as δ_Z^{max}, is given by

$$\delta_Z^{max} = \frac{2(1+\alpha)}{\sqrt{(3+\alpha)(3\alpha+1)}} - 1, \text{ achieved for } \delta_Q^{max} = \sqrt{\frac{3\alpha+1}{3+\alpha}} - 1$$

The associated decrease in emission δ_E^{max} and the maximum difference ($\delta_E^{max} - \delta_Z^{max}$) are respectively given by

$$\delta_E^{max} = \frac{1}{1+\alpha}(1 - \sqrt{\frac{3+\alpha}{3\alpha+1}})(\sqrt{\frac{3\alpha+1}{3+\alpha}} - \alpha), \text{ and}$$

$$\delta_E^{max} - \delta_Z^{max} = \frac{\sqrt{(3+\alpha)(3\alpha+1)}}{2(1+\alpha)}(1 - \sqrt{\frac{3+\alpha}{3\alpha+1}})(\sqrt{\frac{3\alpha+1}{3+\alpha}} - 1).$$

These values can be viewed as maximizing the benefit derived from operational adjustments (the most environmental bang for the cost buck).

Proposition 4 $\delta_E^{max} - \delta_Z^{max}$ *is increasing in* $\alpha > 1$ *and decreasing in* $\alpha < 1$ *and ranges from* 0 *to* $(2 - \sqrt{3}) \approx 26.8\%$.

Proof: It is easy to show that

$$\frac{\partial(\delta_E^{max} - \delta_Z^{max})}{\partial \alpha} = \frac{2(\alpha - 1)}{(1+\alpha)^2\sqrt{(3+\alpha)(3\alpha+1)}} \begin{cases} > 0 & \text{for } \alpha > 1 \\ \\ < 0 & \text{for } \alpha < 1 \end{cases}, \text{ and}$$

$$limit_{\alpha \to 0}(\delta_E^{max} - \delta_Z^{max}) = limit_{\alpha \to \infty}(\delta_E^{max} - \delta_Z^{max}) = 2 - \sqrt{3}. \ \square$$

In the special cases of $\hat{A} = 0$ and $\hat{h} \neq 0$, $\delta_Z^{max} = \frac{2-\sqrt{3}}{\sqrt{3}} \approx 15.5\%$ and $\delta_E^{max} = \frac{\sqrt{3}-1}{\sqrt{3}} \approx 42.3\%$ (in other words, we achieve a 42.3% reduction emissions with only a 15.5% increase in cost).

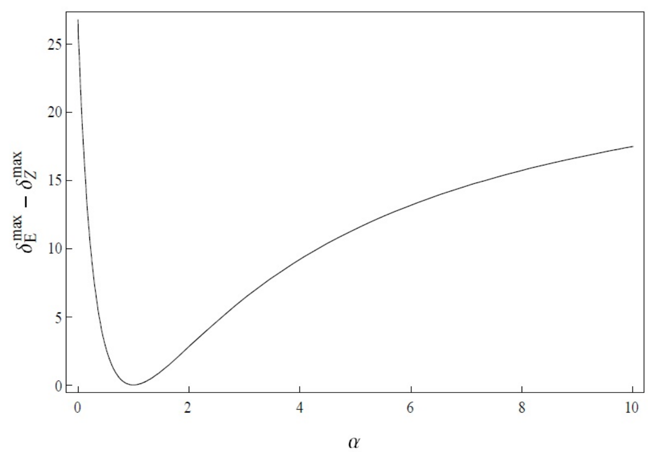


Figure 2.5: The impact of α on the maximum difference between δ_E and δ_Z

We conclude this section by noting that the key results and insights from this section are applicable to settings where either $\hat{A} = 0$ or $\hat{h} = 0$ – that is, settings in which emissions are due only to initiating orders or only to keeping inventory. In both cases, it is easy to see that the opportunity to reduce emissions without significantly increasing cost is even greater; for brevity, we omit the details.

2.3 Extensions to Systems with Carbon Prices

In this section, we briefly describe how the analysis can be extended to settings where emissions are regulated using carbon prices, instead of strict caps, or a combination of caps and prices. In particular, we consider settings with a *carbon tax*, *cap-and-offset*, and *cap-and-price*. In each case, we discuss the extent to which the possibility of an operational adjustment can be leveraged to either reduce emission costs or to generate additional revenue.

2.3.1 Carbon Tax

The pricing of carbon can take on a variety of forms. A simple mechanism is to impose a financial penalty, a tax, per unit of carbon emitted. If we let $t > 0$ denote the penalty per unit of carbon emitted (the tax rate), then the total cost incurred by the firm, given a choice of order quantity Q, can be expressed as follows

$$Z_t(Q) = Z(Q) + tE(Q) \tag{2.10}$$

where,

$$\begin{cases} Z(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD & \text{is the direct operational cost associated with } Q, \\ E(Q) = \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D & \text{is the corresponding emission.} \end{cases}$$

To assess the extent to which operational adjustments can mitigate the cost of emissions we compare two strategies available to a firm. The first is for the firm to continue *business as usual* and to choose the order quantity that minimizes its operational cost $Z(Q)$ and then to pay the resulting tax. This means that the firm chooses order quantity $Q^* = \sqrt{\frac{2AD}{h}}$. The second strategy is for the firm to adjust its order quantity so that it minimizes the sum of its operational and emission costs $Z_t(Q)$. In this case, the firm chooses

$$Q_t^* = \sqrt{\frac{2(A + t\hat{A})D}{(h + t\hat{h})}}. \tag{2.11}$$

Note that $Q_t^* \neq Q^*$ only if $\frac{\hat{A}}{\hat{h}} \neq \frac{A}{h}$. Otherwise, the "operational cost"-optimal solution is also "total cost"-optimal.

Proposition 5 *Let* $\delta_{Z_t'} = \frac{Z_t'(Q^*) - Z_t'(Q_t^*)}{Z_t'(Q^*)}$ *and* $\delta_{E'} = \frac{E'(Q^*) - E'(Q_t^*)}{E'(Q^*)}$ *denote respectively the relative reduction in cost and emission due to the adjustment in the order quantity, where* $Z_t'(Q)$ *and* $E'(Q)$ *refer to the components of cost and emission that are affected by order quantity, then when* $\frac{\hat{A}}{\hat{h}} \neq \frac{A}{h}$

- *Both* $\delta_{Z_t'}$ *and* $\delta_{E'}$ *are positive and strictly increasing in* t *(*$\delta_{E'}$ *is concave in* t*), with* $\delta_{Z_t'} < \delta_{E'}$ *and* $\lim_{t\to\infty} \delta_{Z_t'} = \lim_{t\to\infty} \delta_{E'} = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$*, and*

- *Both* $\delta_{Z_t'}$ *and* $\delta_{E'}$ *are strictly increasing (decreasing) in* $\alpha > 1 (\alpha < 1)$*.*

Proof: First note that $\delta_{Z_t'}$ and $\delta_{E'}$ are respectively given by

$$\delta_{Z_t'} = 1 - \frac{\sqrt{(1+ut)(1+vt)}}{1 + \frac{t}{2}(u+v)} \text{ and } \delta_{E'} = 1 - (\frac{u}{u+v}\sqrt{\frac{1+vt}{1+ut}} + \frac{v}{u+v}\sqrt{\frac{1+ut}{1+vt}}),$$

where $u = \frac{\hat{A}}{A}$, $v = \frac{\hat{h}}{h}$, so that $\alpha = u/v$. Then, we can show that

$$\frac{\partial \delta_{Z_t'}}{\partial t} = \frac{(u-v)^2 t}{\sqrt{(1+ut)(1+vt)}(2+ut+vt)^2} > 0$$

$$\text{and } \begin{cases} \frac{\partial \delta_{E'}}{\partial t} = \frac{(u-v)^2}{2(u+v)((1+ut)(1+vt))^{3/2}} > 0 \\ \\ \frac{\partial^2 \delta_{E'}}{\partial t^2} = -\frac{(u-v)^2(u+v+2uvt)}{4(u+v)((1+ut)(1+vt))^{5/2}} < 0 \end{cases}.$$

Moreover, $Z_t'(Q^*)^2 - Z_t'(Q_t^*)^2 = \frac{AhD}{2}(u-v)^2t^2 > 0$, $E'(Q^*)^2 - E'(Q_t^*)^2 = \frac{AhD}{2}(u-v)^2\frac{(u+v+uvt)t}{(1+ut)(1+vt)} > 0$, and $\delta_{E'} - \delta_{Z_t'} = \frac{(a-b)^2 t}{(a+b)\sqrt{(1+ut)(1+vt)}(2+t(u+v)} > 0$, therefore, $0 < \delta_{Z_t'} < \delta_{E'}$. It is easy to verify that $\lim_{t\to\infty} \delta_{Z_t'} = \lim_{t\to\infty} \delta_{E'} = \frac{(1-\sqrt{\alpha})^2}{1+\alpha}$. Finally, we can show that

$$\begin{cases} \frac{\partial \delta_{E'}}{\partial u} = \frac{(u-v)}{2(u+v)^2}\frac{t(1+vt)(u+3v+2uvt)}{((1+ut)(1+vt))^{3/2}} & > 0(< 0) \text{ for } u > v(u < v), \\ \frac{\partial \delta_{Z_t'}}{\partial u} = \frac{(u-v)t^2}{(2+t(u+v))^2}\sqrt{\frac{1+vt}{1+ut}} & > 0(< 0) \text{ for } u > v(u < v), \end{cases}$$

and

$$\begin{cases} \frac{\partial \delta_{E'}}{\partial v} = \frac{(v-u)}{2(u+v)^2}\frac{t(1+ut)(v+3u+2uvt)}{((1+ut)(1+vt))^{3/2}} & > 0(< 0) \text{ for } u < v(u > v), \\ \frac{\partial \delta_{Z_t'}}{\partial v} = \frac{(v-u)t^2}{(2+t(u+v))^2}\sqrt{\frac{1+vt}{1+ut}} & > 0(< 0) \text{ for } u < v(u > v). \end{cases}$$

Therefore, $\delta_{Z'}$ and $\delta_{E'}$ are strictly increasing (decreasing) in $\alpha > 1(\alpha < 1)$. $\square$

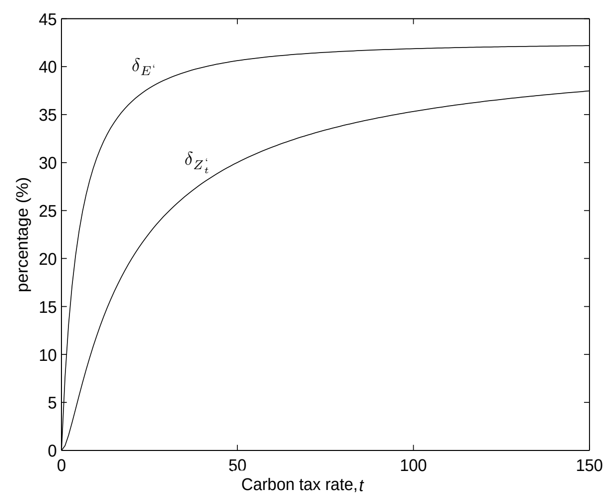


Figure 2.6: The benefit of operational adjustment under carbon tax

The results of Proposition 5 show that operational adjustment can indeed lower a firm's emission cost (its tax burden) and for this reduction to be sufficient to offset the associated increase in its operational cost. The relative benefit from this adjustment (both in terms of cost and emission) increases with the tax rate, as a higher tax rate justifies moving further away from the "operational cost"-optimal order quantity. The fact that the emission reduction is concave in the tax rate implies that there is a diminishing effect to increasing the tax rate; see Figure 2.6. It also implies that a modest tax rate can lead to a significant reduction in emissions. This would be the case for example when α is large (small) for $\alpha > 1(\alpha < 1)$, or equivalently when the absolute difference between the ratios $\frac{A}{h}$ and $\frac{\hat{A}}{\hat{h}}$ is large.

2.3.2 Cap-and-Offset

An alternative to taxing all emissions is to tax only emissions that exceed a certain threshold. This can arise in settings where the regulatory agency sets an emission cap on the firm and imposes penalties if the cap is exceeded. It can also arise in settings where the regulatory agency sets an emission cap but allows firm to *relax* its cap through the purchase of *emission offsets* through third parties (see [5] for examples and references). Note that systems with a strict cap and a tax on all emissions can be viewed as special cases of cap-and-offset, where in the first case the penalty for exceeding the cap is infinitely large and in the second the cap is set at zero.

Under a cap-and-offset system, the firm has again the option of either operating business-as-usual by choosing an order quantity that minimizes the sum of its fixed ordering and inventory holding costs (its direct operational costs) and then to pay any resulting penalties if it exceeds its cap, or of adjusting its order quantity by choosing one that minimizes the sum of its operational and emission costs. In the latter, the problem the firm faces can be formulated as follows:

$$\text{Minimize } Z_o(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD + t(\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} - \hat{C})^+, \tag{2.12}$$

where $\hat{C} = C - \hat{c}D$, t is the penalty paid per unit emitted in excess of the cap, and $X^+ = \max(0, X)$.

Theorem 2 *The optimal order quantity that minimizes (12) is given by*

$$Q_o^* = \begin{cases} Q_t^* = \sqrt{\frac{2(A+t\hat{A})D}{h+t\hat{h}}}, & \hat{C} < \hat{C}_1, \\ \\ Q_1 = \frac{\hat{C}-\sqrt{\hat{C}^2-2\hat{A}\hat{h}D}}{\hat{h}}, & \hat{C}_1 < \hat{C} < \hat{C}_2 \textit{ and } \frac{A}{h} < \frac{\hat{A}}{\hat{h}}, \\ \\ Q_2 = \frac{\hat{C}+\sqrt{\hat{C}^2-2\hat{A}\hat{h}D}}{\hat{h}}, & \hat{C}_1 < \hat{C} < \hat{C}_2 \textit{ and } \frac{A}{h} > \frac{\hat{A}}{\hat{h}}, \\ \\ Q^* = \sqrt{\frac{2AD}{h}}, & \hat{C} > \hat{C}_2, \end{cases}$$

where $\hat{C}_1 = \sqrt{\frac{D}{2}}(\hat{A}\sqrt{\frac{h+t\hat{h}}{A+t\hat{A}}} + \hat{h}\sqrt{\frac{A+t\hat{A}}{h+t\hat{h}}})$, *and* $\hat{C}_2 = \sqrt{\frac{D}{2}}(\hat{A}\sqrt{\frac{h}{A}} + \hat{h}\sqrt{\frac{A}{h}})$.

Proof: $\hat{C}_1$ and $\hat{C}_2$ correspond to $E(Q_t) - \hat{c}D$, $E(Q^*) - \hat{c}D$ respectively. When $\hat{C} \geq \hat{C}_2$, we know that for any Q, $Z_o(Q^*) \leq Z_o(Q)$, therefore $Q_o^* = Q^*$. When $\hat{C}_1 \leq \hat{C} \leq \hat{C}_2$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we know that for any Q satisfying $E(Q) \geq C$, $Q \leq Q_1 \leq Q_t$. Therefore, $Z_o(Q_1) \leq Z_o(Q)$. On the other hand, for any Q satisfying $E(Q) < C$, we know that $Q^* \leq Q_1 \leq Q$. Therefore, $Z_o(Q_1) \leq Z_o(Q)$. Therefore, when $\hat{C}_1 \leq C \leq \hat{C}_2$, we have $Q_o^* = Q_1$. The case of $\hat{C}_1 \leq C \leq \hat{C}_2$ and $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ is similar. When $\hat{C} \leq \hat{C}_1$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we know that for any Q satisfying $E(Q) \geq C$, $Z_o(Q_t^*) \leq Z_o(Q)$. On the other hand, for any Q satisfying $E(Q) < C$, we know that

$Q > Q_1 > Q_t^*$, because $\hat{C} < \hat{C}_1, \frac{A}{h} < \frac{\hat{A}}{\hat{h}}$. Therefore, $Z_o(Q) > Z_o(Q_1) > Z_o(Q_t^*)$. Thus, when $\hat{C} < \hat{C}_1$ and $\frac{A}{h} < \frac{\hat{A}}{\hat{h}}$, we have $Q_o^* = Q_t^*$. The case of $\hat{C} < \hat{C}_1$ and $\frac{A}{h} > \frac{\hat{A}}{\hat{h}}$ is similar. □

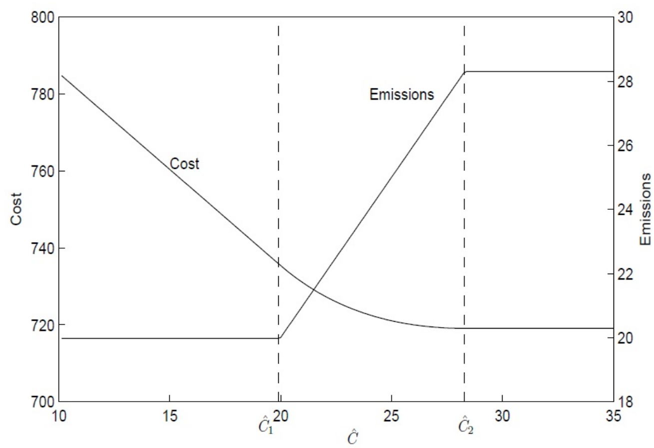


Figure 2.7: The effect of the carbon cap under cap-and-offset
$(D = 100, A = 120, h = 2, c = 5, \hat{A} = 1, \hat{h} = 0.5, \hat{c} = 0, t = 5)$

Theorem 2 indicates that, depending on the cap, there are three regions of operation. If the cap is sufficiently large (i.e., $\hat{C} \geq \hat{C}_2$), then the firm operates business-as-usual and does not make any adjustments in its order quantity. If $\hat{C}_1 \leq \hat{C} \leq \hat{C}_2$, the firm makes adjustments in its order quantity so that its emissions do not exceed the cap; the firm in this case does not incur any emission penalties. Note that here the firm operates as if it were subject to a strict emission cap and therefore all the results of Section 2 apply, including the opportunity to significantly reduce emissions without significantly increasing cost. Note also that in this region the firm finds it more preferable to incur the increase in its direct operational costs than to incur emission penalties. On the other hand, if

$\hat{C} \leq \hat{C}_1$, then the firm finds it more preferable to pay the emission penalty than to reduce its emissions below $C_1 = \hat{C}_1 + \hat{c}D$. In this region, the firm emits exactly C_1 and pays the penalty $t(C - C_1)$ associated with the difference. The three regions are illustrated for an example system in Figure 2.12. Note that for the strategy of not adjusting order quantity, cost is linear in the cap for $\hat{C} \leq \hat{C}_2$. Hence, the difference in cost between adjusting and not adjusting order quantity can be significant in the region where $\hat{C}_1 \leq \hat{C} \leq \hat{C}_2$. It can also be significant when $\hat{C} \leq \hat{C}_1$, where the difference is constant and equal to $t(C_2 - C_1)$.

We conclude this section by highlighting the effect of the emission penalty t. In particular, an increase in t has the effect of decreasing the value of $\hat{C}_1$ and, therefore, increasing the width of the interval $[\hat{C}_1, \hat{C}_2]$. This means that a larger t increases the region in which incurring higher operational costs is preferable to incurring emission penalties.

2.3.3 Cap-and-Price

Instead of only penalizing emissions that exceed the specified cap, as in cap-and-offset, the regulating agency may choose to also encourage emissions that are lower than the cap by rewarding firms that emit less than their cap. Rewarding lower emissions can take on a variety of forms. At its simplest, it consists of a reward (penalty) t' (t) per unit of emission below (above) the cap C. We refer to this scheme as cap-and-price since there is now monetary value for emissions regardless of whether or not the cap is exceeded (this is similar to the so-called *cap-and-trade* system, except that in that case prices are established through a market mechanism for carbon trading). All the previous schemes can be viewed as special cases of cap-and-price (for example cap-and-offset corresponds to the case where $t' = 0$).

It is easy to show that when $t' = t$ the problem is similar to one with a carbon tax, with a similar expression for total cost except for an additional revenue term $-tC$ (this extra term is due to the fact that cap-and-price is equivalent to a scheme where firms receive a reward equal to tC and are then taxed for each unit of emission with rate t). Therefore, many of the results obtained for the case of carbon tax continue to apply. However, in contrast to a system with a carbon tax, the effect of t on cost is not monotonic. In particular, if the cap C is sufficiently large (larger than the emission that would be incurred if the firm maintained business as usual), then cost would be strictly decreasing in t, eventually becoming negative with the firm realizing a net profit. On the other hand, if the cap is sufficiently small (smaller than the emission that would be incurred if the firm chose the emission-optimal order quantity), then cost would be strictly increasing in t since the amount of emissions would always exceed the cap. If the cap falls between these two thresholds, then cost is not monotonic in t; it initially increases then decreases. In particular, when t is small, the firm chooses to exceed the emission cap and, therefore, incurs the corresponding emission penalties. However, when t is large, the firm chooses to emit less than the cap and realizes the corresponding revenue. This effect is illustrated in Figure 2.8. Note that the benefit from adjusting the order quantity to take acount of carbon price can be significant, in terms of both cost and emission, when carbon price is high and the firm can emit less than its cap.

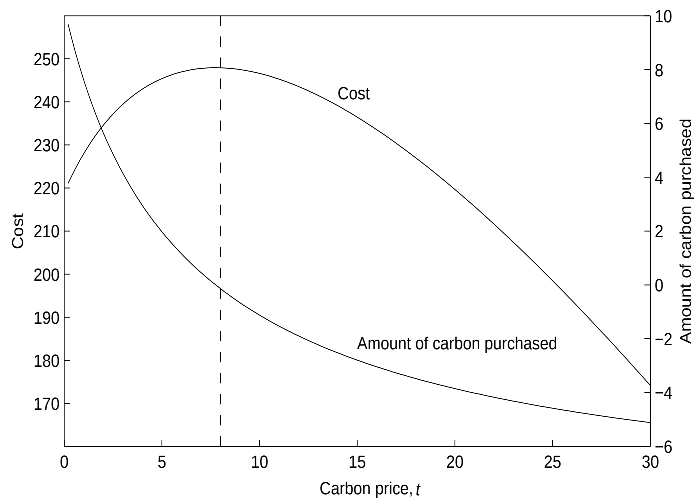


Figure 2.8: The impact of carbon price on cost and carbon emissions purchased when $E_{min} \leq C \leq E_{max}$
$(D = 100, A = 120, h = 2, \hat{A} = 1, \hat{h} = 0.5, c = 0, \hat{c} = 0)$

2.4 Extensions to Other Models

In the Section 2, we showed how it may be possible to significantly reduce emissions without significantly increasing cost via only an operational adjustment. In particular, we showed that the key requirements are that the cost and the emission functions yield different optimal solutions and that the cost function exhibits less sensitivity around the cost-optimal solution than the emission function. These features can be shown to be present in several other operational settings and models. Therefore, the insights obtained have applicability to those cases as well. In what follows, we briefly describe two other widely used operational models for which this is the case.

2.4.1 The Facility Location Model

Consider the problem of determining the number of facilities N to serve demands that are uniformly distributed over a region with an area a and demand density ρ (amount of demand per unit of area). There is a fixed cost f incurred with each facility and a variable cost c incurred with each unit of demand transported one unit of distance. There is a tradeoff between the fixed costs and the transportation costs, with a higher number of facilities leading to higher fixed costs but, because the service area of each facility is a/N, lower transportation costs. Under the assumptions that a is a square area and travel occurs along paths oriented at 45 degrees to the sides of the square, the total expected cost $Z(N)$ can be shown to be given by (see for example [19] and [20])[1]

$$Z(N) = fN + c\rho a \frac{2}{3}\sqrt{\frac{a}{2N}}. \tag{2.13}$$

Similarly to cost, there is a fixed emission $\hat{f}$ associated with each facility and variable emission $\hat{c}$ associated with each unit of demand transported one unit of distance, leading to an expected total emission $E(N)$ given by

$$E(N) = \hat{f}N + \hat{c}\rho a \frac{2}{3}\sqrt{\frac{a}{2N}}. \tag{2.14}$$

We can then show that the cost-optimal and emission-optimal number of facilities are given by $N^* = K(\frac{c}{f})^{2/3}$ and $\hat{N}^* = K(\frac{\hat{c}}{\hat{f}})^{2/3}$, respectively, where $K = a(\frac{\rho}{2\sqrt{3}})^{2/3}$.

As in the EOQ case, there is an opportunity to reduce emissions via an adjustment in the number of facilities whenever $\frac{\hat{c}}{\hat{f}} \neq \frac{c}{f}$. Moreover, we can show that the cost function is flat around the optimal solution and that whenever the flat region

[1] In general, $Z(N) = fN + \frac{c}{\sqrt{N}}g(a)$, where $g(a)$ is a function dependent on the shape of area a.

of the cost function coincides with the steep region of the emission function, a significant reduction in emissions can be achieved without a significant increase in cost. To see this let $\alpha = \frac{\hat{c}}{\hat{f}} / \frac{c}{f}$, $\delta_N = \frac{N-N^*}{N^*}$, $\delta_Z = \frac{Z(N)-Z(N^*)}{Z(N^*)}$, and $\delta_E = \frac{E(N^*)-E(N)}{E(N^*)}$. Then, it is easy to verify that,

$$\delta_Z = \frac{1}{3}(\delta_N - 2 + \frac{2}{\sqrt{1+\delta_N}}) \text{ and } \delta_E = \frac{1}{1+2\alpha}(1 + \delta_N + \frac{2\alpha}{\sqrt{1+\delta_N}}).$$

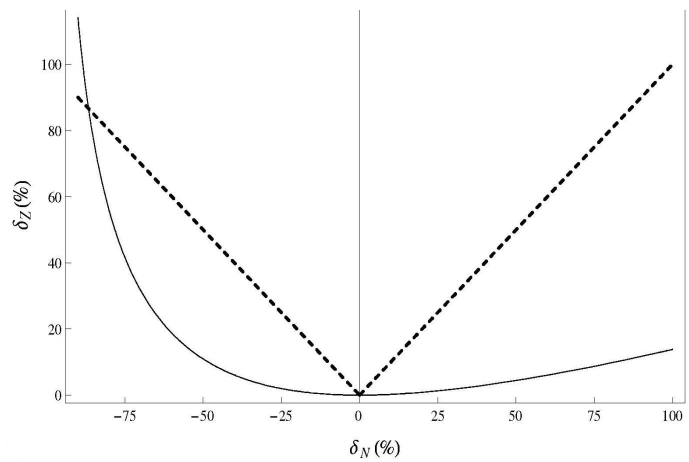


Figure 2.9: The impact of changes in the number of facilities on cost

The flatness of the cost function is apparent from Figure 2.9 (e.g., a 50% increase in N leads to less than 5% increase in cost). Mathematically, the cost ratio δ_Z is strictly convex in δ_N with $\frac{\partial \delta_Z}{\partial \delta_N}(0) = 0$. In contrast, the emission ratio δ_E is strictly concave in δ_N with $\frac{\partial \delta_E}{\partial \delta_N}(0) = \frac{\alpha-1}{1+2\alpha}$, which is increasing (decreasing) in $\alpha > 1$ ($\alpha < 1$) (recall that α is increasing in the absolute value of the difference $\frac{\hat{c}}{\hat{f}} - \frac{c}{f}$). Moreover,

- $\delta_Z \leq \delta_E$, for $|\delta_N| \leq |\frac{1}{2}(\frac{9\alpha}{2+\alpha} - \sqrt{\frac{3(2+7\alpha)}{2+\alpha}})|$, and
- $\delta_E - \delta_Z$ is maximal for $\delta_N^{max} = (\frac{1+5\alpha}{4+2\alpha})^{2/3} - 1$.

In the limit case of $\alpha \to 0$, $\delta_E(\delta_N^{max}) - \delta_Z(\delta_N^{max}) \to 2 - 2^{2/3} \approx 0.412$ (we achieve approximately 60% reduction in emissions with only about 19% increase in cost) and in the limit case of $\alpha \to \infty$, $\delta_E - \delta_Z \to 2 - (5/2)^{2/3} \approx 0.15$ (we achieve approximately 26% reduction in emissions with only about 10% increase in cost).

2.4.2 The Newsvendor Model

Consider the problem of determining the order quantity Q for a single selling period given that demand for the period is stochastic and can be described by a continuous random variable D. Ordering too little could lead to shortages while ordering too much could lead to leftover inventory, with a shortage cost c_s incurred per unit of demand that exceeds the quantity ordered Q and an overage cost c_o incurred per unit ordered that exceeds demand. Thus, the total expected cost for the period is given by

$$Z(Q) = c_s\mathbb{E}[(D - Q)^+] + c_o\mathbb{E}[(Q - D)^+], \tag{2.15}$$

where $\mathbb{E}[.]$ refers to the expected value operator. Similarly, there are emissions associated with being short and with being over. In particular, $\hat{c}_s$ is the amount of emissions per unit of shortage (e.g., emissions caused by resorting to fulfilling demand through more emission-intensive means) and $\hat{c}_o$ is the amount of emissions per unit of overage (e.g., emissions associated with disposing of leftover inventory or with storing inventory until the next selling season). This leads to a total expected emission given by

$$E(Q) = \hat{c}_s\mathbb{E}[(D - Q)^+] + \hat{c}_o\mathbb{E}[(Q - D)^+]. \tag{2.16}$$

Then, it is easy to show that the cost-optimal order quantity Q^* is the solution to the critical fractile equation (see for example [58]): $F(Q) = \frac{c_s}{c_s+c_o}$, and the emission-optimal order quantity $\hat{Q}^*$ is given by the solution to $F(Q) = \frac{\hat{c_s}}{\hat{c_s}+\hat{c_o}}$, where F is the probability distribution of demand D.

As with the EOQ and the facility location models, there is an opportunity to reduce emissions by modifying the order quantity whenever $c_s/c_o \neq \hat{c}_s/\hat{c}_o$. For a variety of demand distributions, it is possible to show that the cost function is flat around the cost-optimal solution while the emission function is steep in the same region, leading to the possibility of significantly reducing emissions without significantly increasing cost.

Consider for example the case where demand is uniformly distributed over an interval $[a, b]$, with $b > a \geq 0$. We can easily verify that the cost-optimal order quantity is given by $Q^* = \frac{ac_o+bc_s}{c_o+c_s}$, the corresponding expected cost $Z(Q^*)$ and emission level $E(Q^*)$ are respectively given by

$$Z(Q^*) = \frac{c_s c_o}{2(c_s + c_o)}(b-a) \text{ and } E(Q^*) = (b-a)\frac{c_s^2\hat{c}_o + c_o^2\hat{c}_s}{2(c_s+c_o)^2}. \tag{2.17}$$

We can show that δ_Z is strictly convex in δ_Q while δ_E is strictly concave in δ_Q, with $\frac{\partial\delta_Z}{\partial\delta_Q}(0) = 0$, and $\frac{\partial\delta_E}{\partial\delta_Q}(0) \neq 0$.

Moreover, we can show that the maximum value δ_Q^0 by which the order quantity can be adjusted (increased (decreased) when $\alpha > 1(\alpha < 1)$) while guaranteeing that $0 \leq \delta_Z \leq \delta_E$ is given by $\delta_Q^0 = \mid \frac{4(a-b)(1-\alpha)}{\frac{c_o}{c_s}+1(a+b\frac{c_s}{c_o})(1+\alpha)} \mid$, where $\alpha = \frac{\hat{c_s}/\hat{c_o}}{c_s/c_o}$. We can also show that δ_Q^{max}, the relative change in the order quantity for which the difference $(\delta_E - \delta_Z)$ is maximum, is given by $\delta_Q^{max} = \frac{2(a-b)(1-\alpha)}{\frac{c_o}{c_s}+1(a+b\frac{c_s}{c_o})(1+\alpha)}$. When the order quantity is adjusted by δ_Q^{max}, we can easily verify that, $\delta_E(\delta_Q^{max}) = (2 + \frac{\alpha}{1+\frac{c_o}{c_u}\alpha})\delta_Z(\delta_Q^{max})$. This means that, independently of the parameters of the demand distribution, adjusting the order quantity by δ_Q^{max} leads to a reduction in emissions that is at least twice the increase in cost.

2.5 Social Welfare Analysis

In this paper, we have assumed that the parameters of the emission regulation are exogenous. It is however possible to take the perspective of a social planner who decides on the parameters of the regulation to maximize social welfare. We define social welfare (SW) as the difference between the sum of producer surplus (PS), consumer surplus (CS) and tax revenue (TR) on one hand and environmental damage (ED) on the other, so that $SW = PS + CS + TR - ED$. For the sake of brevity we consider only systems operating under a strict emission cap or an emission tax. A similar analysis can be carried out for other policies.

In this paper, we assume that both demand and selling prices are fixed. This assumption is consistent with a firm that is a price taker (e.g., a firm that does not have the scale to affect the prevailing price). Consequently, consumer surplus is fixed and remains the same regardless of the emission regulation policy. Similarly, the revenue component of the producer (selling price multiplied by demand) is fixed. Therefore, the surplus of the producer is determined only by its operational cost in the case of a strict emission cap and by the sum of its operational cost and tax payments in the case of an emission tax. Therefore, social welfare reduce to the difference between the producer's cost and environmental damage.

Consider first the case of a carbon tax. In this case, the social planner's objective is to choose a tax rate t that maximizes social welfare. Given a tax rate t, the firm's optimal order quantity is $\tilde{Q} = \sqrt{\frac{2\tilde{A}D}{\tilde{h}}}$. Letting w denote the estimated environmental cost per unit of emission, maximizing the social welfare is equivalent to

$$\underset{t}{\text{Minimize}} \; \frac{AD}{\tilde{Q}} + \frac{h\tilde{Q}}{2} + w(\frac{\hat{A}D}{\tilde{Q}} + \frac{\hat{h}\tilde{Q}}{2}), \tag{2.18}$$

Let $\tilde{\tilde{A}} = A + w\hat{A}$, $\tilde{\tilde{h}} = h + w\hat{h}$, and $\tilde{\tilde{Q}} = \sqrt{\frac{2\tilde{\tilde{A}}D}{\tilde{\tilde{h}}}}$. Then, it is easy to see that $t = w$ maximizes social welfare, with a corresponding emission level given by $E(\tilde{\tilde{Q}})$ and a producer cost given by $\frac{AD}{\tilde{\tilde{Q}}} + \frac{h\tilde{\tilde{Q}}}{2} + tE(\tilde{\tilde{Q}})$.

Consider now the case of a strict emission cap. In this case, the social planner's objective is to choose an emission cap C that maximizes social welfare. The producer's optimal order quantity, denoted by $\hat{Q}^*$, is either Q^* or $Q_{1,2}$ depending on the carbon cap C. Therefore, maximizing the social welfare is equivalent to

$$\underset{C}{\text{Minimize}} \; \frac{AD}{\hat{Q}^*} + \frac{h\hat{Q}^*}{2} + w(\frac{\hat{A}D}{\hat{Q}^*} + \frac{\hat{h}\hat{Q}^*}{2}). \tag{2.19}$$

It is easy to see that social welfare is maximized when $\hat{Q}^* = \tilde{\tilde{Q}}$, which can be achieved by setting the optimal carbon cap such that $C^* = E(\tilde{\tilde{Q}})$. Therefore, under the optimal strict emission cap policy, the emission level is $E(\tilde{\tilde{Q}})$ and the corresponding optimal social welfare is the same as the one achieved under the optimal tax policy. The producer surplus is however higher since the producer incurs the same operational cost but makes no tax payments.

2.6 Conclusion

In this paper, we provide analytical support for the notion that it may be possible, via operational adjustments alone, to significantly reduce emissions without significantly increasing cost. Using the EOQ model, we provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and discuss factors that affect the difference in the magnitude of emission reduction and cost increase. We then show that the treatment can be easily extended to other operational settings. We illustrate the broader applicability of our key results to two other important classes of operational models, namely facility location modelsmodels, where costs and emissions are driven by a fixed component and a distance-dependent component; and newsvendor models, where costs and emissions are driven by inventory overages and inventory shortages. In both cases, we show that the cost function is remarkably flat around the optimal, providing an opportunity under some conditions to significantly reduce emission without significantly increasing cost.

We also take the perspective of a social planner who decides on the parameters of the regulation to maximize social welfare. We show that for the setting described in this paper, a strict emission cap and an emission tax, when designed optimally, both achieve the same level of social welfare, including the same level of emissions. However, a strict emission cap is preferred by the producer since it does not involve any tax payments.

Chapter 3

On the Effectiveness of Emission Penalties in Decentralized Supply Chains

3.1 Introduction

There is growing pressure on governments and firms worldwide to reduce the emission of harmful pollutants, such as carbon dioxide, or the consumption of scarce natural resources, such as water, to levels that are deemed environmentally sustainable. Imposing penalties on emissions[1] is one of the mechanisms adopted by several countries to achieve these goals. These penalties can be direct in the form of taxes imposed on the emissions of individual firms or facilities, or indirect

[1] For the remainder of the paper, we will use the term emission exclusively. However, our analysis extends to the production of negative externalities, broadly defined, including the consumption of scarce resources.

because of limits (caps) imposed on the amount a firm or a facility can emit. In the case of the latter, the penalties correspond to the marginal cost of meeting emission caps, the price of purchasing emission *offsets* (offsets are reductions in emissions in one place that can be used to compensate for emissions elsewhere), or the prevailing market price of tradable emission permits in cap and trade-like systems.

Imposing penalties on emissions is also a mechanism that an increasing number of firms have adopted voluntarily in their efforts to mitigate their emissions. For example, several firms have committed to becoming *carbon-neutral* through the purchase of carbon offsets (a carbon-neutral firm is one that is credited with reducing global emissions by an amount equal to the one it produces). In order to pay for these offsets, some firms, such as Microsoft and Disney, are imposing internal taxes on their business units for the carbon emissions for which these business units are directly responsible; see DiCaprio (2013) and Davenport (2013). Other firms are putting a price on emissions in evaluating the attractiveness of new investments or to justify investments in conservation projects or renewable energy. A recent report by CDP found at least 30 companies, ranging from utilities, such as Xcel Energy, to energy companies, such as Exxon, and retailers, such as Walmart, setting an internal price ranging from $6 to $60 per metric ton on their carbon pollution (see CDP (2013a)). Firms that are water-intensive, such as Coca Cola, are putting a price on water that is substantially higher than the prevailing price (see Fellow (2013)).

Clearly, everything else remaining the same, imposing a penalty on emissions makes *business-as-usual* more expensive and provides an incentive for firms and business units to reduce their emissions. Thus, it is often argued that such penalties should lead (through the emission reduction efforts of firms or business units) to an economy-wide (or a firm-wide) reduction in total emissions. In this paper, we examine the validity of this premise in the context of a decentralized supply chain (a supply chain consisting of independent firms or of independent business units within a single firm). Perhaps surprisingly, we find that penalizing each firm for the emissions for which it is directly responsible can paradoxically lead to higher overall supply chain emissions and for these emissions to increase in the emission penalty. We show that this paradox arises because firms, in their efforts to reduce their own emissions, can make decisions that adversely impact the costs and emissions of their suppliers or customers.

We base our analysis on a buyer-supplier model that has been extensively studied in the supply chain coordination literature, where the buyer and the supplier make decisions regarding the size of order replenishments. We show how the buyer can choose order sizes that reduce its own emissions but increase the emission of the supplier, with the net effect being an increase in total supply chain emissions and we provide necessary and sufficient conditions under which this occurs. We examine how this effect could arise in other settings and discuss two such examples. The first involves a make versus buy problem, where a manufacturer decides on how much to produce in-house and how much to outsource. The second involves a retail store density problem, where a retailer decides on how many stores to operate in a region. Then, we describe a general model of which the various models we discuss can be viewed as special cases. We also provide a general necessary and sufficient condition for when higher emission penalties lead to higher supply chain emissions.

Remedies that can mitigate the potential negative impact of emission penalties require that one firm internalizes either partially or fully the cost of the externality it imposes on the other firm. We study three such remedies. The first involves the firms coordinating their ordering decisions, either explicitly or implicitly, so that total supply chain profit is maximized. The second is one where both the buyer and the supplier are penalized for a fraction of total supply chain emissions instead of only the emissions for which they are directly responsible. The third involves penalizing consumers instead of the firms. We show that while all three remedies guarantee that higher penalties lead to lower supply chain emissions, the resulting social welfare can be different. More significantly, none of the schemes guarantees that, for a given penalty, emissions would be lower than those obtained when firms are penalized only for their own emissions.

To our knowledge, our paper is among the first to highlight the possibility for emission penalties (and, more generally, emission reduction efforts) to *backfire* because of supply chain interactions. Much of the existing literature in both operations management and environmental economics treats cases where the costs and emissions of one firm are unaffected by the emission mitigation efforts of another firm, The recognition that this may not be the case could have implications in how either policy makers or firms go about designing schemes to reduce total supply chain emissions and in how emission responsibility should be allocated across the supply chain. Our results also highlight the importance of understanding supply chain interactions in assessing the effectiveness of different approaches to emission mitigation.

Our work is of course not the first to point out that well-intentioned actions can lead to undesirable consequences. In the area of energy conservation, there are several well-known paradoxes. *Jevons' paradox* (or the *rebound* effect) refers to improvements in energy efficiency leading to more energy consumption not less (see for example Alcott (2005) and Jevons (1865)). For example, as air conditioning becomes more energy-efficient, more residential and office buildings install air conditioning, leading to an increase in air conditioning-related energy consumption and not less. The *indirect rebound effect* refers to savings from greater efficiency in one area leading to increased consumption in other more emission-intensive areas (see for example Greening et al. (2000) and Sorell (2009)). For example, improved home insulation leads to less household spending on heating, which in turn leads to more spending on other goods and services that may be more energy intensive (e.g., air travel). The *global rebound* effect refers to reduction in energy consumption in one country leading to lower global prices triggering more consumption in other countries (see for example Sorrell and Dimitropoulos (2008)). The *leakage effect* refers to pollution regulation in one country leading to production moving to another country where production is more emission-intensive and regulation is more lax (see for example Babiker (2005)). The effect we describe in this paper appears to be distinct from those described above and caused by the fact that costs and emissions of one firm can be affected by the actions of another.

The rest of the paper is organized as follows. In Section 3.2, we provide a brief review of related literature. In Section 3.3, we describe the buyer-supplier model and specify conditions under which higher emission penalties lead to higher supply chain emissions. In Section 3.4, we discuss potential remedies. In Section 3.5, we extend our analysis to the case where demand is endogenous. In Section 3.6, we discuss other examples and a general model. In Section 3.7, we offer concluding comments.

3.2 Related Literature

The literature on supply chain management is extensive, including literature that deals explicitly with the misalignment of incentives among firms within the same supply chain; see for example Cachon and Lariviere (2005) and Cachon (1998) for reviews. This literature is primarily focused on identifying contracting mechanisms that coordinate the supply chain and ensure that supply chain profit is maximized. (In this paper, we show that coordination does not necessarily result in less emission than no-coordination; with coordination, firms may choose to emit more if it leads to higher profits.) There is also growing literature that is concerned with issues of sustainability in supply chains; see for example Drake and Spinler (2013), Dekker and Bloemhof (2012), and Tang and Zhou (2012) for recent reviews. Within this literature, papers that explicitly consider emissions are relatively few. We review some of the most relevant ones below. More extensive reviews can be found in Plambeck (2012), Benjaafar et al. (2013), and Cachon (2014).

Caro et al. (2013) consider a setting where multiple firms contribute to total supply chain emissions. They address the question of how to allocate emission responsibility among the firms and show that over-allocating emissions, or *double-counting*, is necessary for firms to exert their first-best emission reduction efforts. An important difference between our work and that of Caro et al. is that in their case the emission reduction, or *abatement*, costs of one firm are not affected by the emission reduction effort of another. In our case, this interaction is central to the results we describe (see also the discussion in Section 3.6.3).

Granot et al. (2014) study emission allocation in a setting where firms are accountable for their own emissions as well as the emissions from all upstream suppliers. Using a cooperative game theory approach, they show that an emission responsibility sharing scheme where the direct emission of one firm is allocated equally among all its downstream firms is in the core and corresponds to the Shapley value. Their analysis relies on the assumption that, absent emission sharing, firms are held accountable for the emissions of all their upstream suppliers. They also assume that emissions are exogenously specified and not affected by how emission responsibility is allocated and by the corresponding actions of individual firms. In this paper, we consider settings where firms modify their actions in response to allocations and these actions can affect the costs and emissions of all the firms in the supply chain.

Cachon (2014) considers the problem of a retailer deciding on the number and size of stores taking into account the costs and emissions incurred by both the retailer and the consumers. He shows that an emission tax would have to be relatively high to affect the design of the retail network. In contrast to our setting of a decentralized supply chain, Cachon considers a setting where the retailer acts as a central planner and fully compensates consumers for their travel and emission costs. In Section 3.6.2, we revisit the model in Cachon (2014) and show that, if the retailer and the consumers act independently, then the retailer's effort to reduce its own emissions can lead to higher consumer emissions, which in turn can lead to higher overall emissions.

Sunar and Plambeck (2014) consider the impact of a cross border adjustment tax (a tax on the emissions caused by a product's creation and transportation to the border) incurred by an importer. They show that, in the presence of co-products and for a specific type of allocation of emission among the products, it is possible for a higher tax to lead to higher emissions. This is due to the fact that higher taxes can lead the supplier of the imported good to lower its selling price resulting in more demand and therefore more emissions.

Benjaafar et al. (2013) show how carbon emission parameters could be incorporated in lot sizing models. They investigate numerically the impact of imposing an emission cap on supply chain cost and observe that there are cases where emissions can be significantly reduced without significantly increasing cost. Chen et al. (2013) validate these numerical observations for a setting with a single firm using the EOQ model. Subramanian et al. (2007) study abatement investment in a deterministic oligopoly where the price of emissions is endogenous. Velázquez-Martínez et al. (2014) compare different approaches to carbon footprint calculations in supply chains and show that carbon emission aggregation can lead to poor estimation of actual footprint. Hoen et al. (2014) consider carbon emissions in transportation and find that accounting for emission cost would have only a limited effect on the relative desirability of different transportation modes.

Drake (2012) studies the effect of carbon trading on carbon leakage and domestic and foreign firms' comparative advantage. Drake et al. (2012) consider capacity investment and production decisions under both a cap-and-trade system (where carbon price is stochastic) and a carbon tax (where carbon price is fixed) and find that expected profits are greater, and expected emissions are lower under cap-and-trade. Krass et al. (2013) find that the choice of green technology in response to an increase in emission taxes may be non-monotone. Gong and Zhou (2013) analyze the joint production control and emission trading problem in the presence of alternative production technologies.

There is also significant research that draws on the methodology of life cycle analysis (LCA) to document carbon emissions that can be attributed to a product at various stages of its production, distribution, consumption, and end-of-life disposal; see for example Reap et al. (2008) and Matthews et al. (2008). An important stream in this literature uses economic input-output (EIO) modeling to account for carbon emissions at a sector level in the economy and to study the impact of one sector on the carbon emissions of other sectors; see Wiedmann (2009).

Finally, there is extensive literature from environmental economics that examines the optimal design of environmental policies from the perspective of a social planner (see for example Baumol (1972), Barnett (1980), and Katsoulacos and Xepapadeas (1995)). Results from this literature show that taxing emissions always increases abatement effort and reduces emissions. They also show that a *Pigouvian* tax (taxing emissions using the true social cost of emissions) is socially-optimal under perfect competition but may not be otherwise (e.g., imposing a Pigouvian tax on a monopolist is not socially optimal and could hurt consumption by increasing price and shrinking demand). This literature does not account for the supply chain dynamics we model in this paper. In particular, it assumes that the abatement effort of one firm does not affect the cost of abatement of other firms. As we discuss in Section 5, when supply chain interactions are accounted for, the socially-optimal tax can be either lower or higher than a Pigouvian tax.

3.3 A Buyer-Supplier Model

We consider a buyer-supplier model that is perhaps the most widely studied model of supply chain interactions, with literature that spans more than three decades; see for example Jeuland and Shugan (1983), Lal and Staelin (1984), Weng (1995), Corbett and De Groote (2000), Cachon (2003), Cachon and Kok (2010) and the references therein. The model is popular because of its ability to capture important supply chain dynamics, the robustness of the results it yields despite of its succinctness, and because of its mathematical tractability. Specifically, we consider a serial supply chain consisting of a buyer (Firm 1) and a supplier (Firm 2). The buyer faces external demand, constant with a fixed rate that may depend on the selling price, and decides on the frequency with which to order from the supplier (or equivalently on the order quantities). In doing so, the buyer trades off fixed ordering costs with variable inventory holding costs. The supplier, in response to the orders from the buyer, decides similarly on the frequency of ordering from its own supplier (or on the frequency of producing internally) trading off fixed ordering (or production setup) costs with inventory holding costs.

Let D denote demand per unit time and p denote the unit selling price (we treat demand as being exogenous for the time being and relax it later in Section 5 by making price a decision variable). The buyer incurs a fixed ordering cost

A_1 per order, a holding cost h_1 per unit kept in inventory per unit time, and a variable cost c_1 per unit purchased from the supplier in addition to the unit price w the supplier charges. The supplier incurs similar costs with corresponding parameters A_2, h_2, and c_2, respectively. Without loss of generality, we assume that orders are delivered from the supplier to the buyer with zero leadtime (a positive leadtime can be included and does not affect the solution to the problem); we also assume that each firm must always keep enough inventory to satisfy all the demand from its customers (the analysis can be easily extended to settings with backorders). It is possible to consider more general cost functions, including when cost parameters are non-linear or exhibit discontinuities. However, the qualitative insights we derive remain the same and we keep, consistent with much of the existing literature, the simpler form for the sake of tractability.

In addition to costs, there may be emissions associated with ordering, processing, and inventory holding. We let $\hat{A}_i$ denote the amount of emissions associated with each order placed by firm i (e.g., due to transportation or process setup), $\hat{h}_i$ the amount of emissions due to a unit held in inventory per unit time (e.g., due to heating or refrigeration of stored inventory), and $\hat{c}_i$ the amount of emissions per unit purchased or produced (e.g., due to energy consumed during handling and processing). We only require that these parameters be non-negative so that it is possible for some to be zero (for example, a firm may have negligible emissions associated with initiating orders, holding inventory, or with order processing). Similar to cost, it is possible to consider more general emission functions and the results we describe continue to hold when the emission parameters exhibit non-linearities (see for example, Section 3.6.1).

Similar to costs, characterizing emission parameters in practice requires a data collection and documentation effort. Fortunately, emission data is increasingly available on standard processes, including transportation and storage; see for example Cachon (2014) and Chen et al. (2013). There are also increasingly accepted standards for measuring and reporting different categories of emissions, or so-called emission scopes[2] .

We consider a setting where each firm incurs a penalty at rate t per unit of emissions it emits (scope 1 and 2 emissions). As discussed in the introduction, the penalty can be self imposed or externally mandated. In the case where the penalty is self imposed, the rate t may correspond to the prevailing market price of offsets. If it is externally mandated, it may correspond to a tax imposed by a regulating entity. It is possible to consider settings where the penalty rate is non-linear and varies with the amount of emissions. It is also possible to consider settings where only one of the firms incurs an emission penalty (e.g., the case where one firm chooses to offset its emissions but not the other) or where the emission penalties are different for different firms. In what follows, we treat the case where both firms are subject to the same penalty (e.g., both firms seek to be emission-neutral by purchasing offsets from the same offset market). However, the effects we describe are also present in the more general settings mentioned above (see Section 6.2 for an example where only one supply chain party is subject to an emission penalty).

[2] A prominent example of such a standard is the Green House Gas (GHG) Protocol developed by the World Resource Institute and the World Business Council for Sustainable Development. According to the GHG protocol, emissions are classified into scope 1, scope 2, and scope 3 emissions, with scope 1 referring to direct emissions by the firm at its installations, scope 2 to indirect emissions by the firm via electricity usage, and scope 3 to emissions by the firms' upstream and downstream supply chains (GHG Protocol (2011)).

The objective of each firm is to maximize its own after penalty profit by choosing an ordering quantity, q_i for firm $i = 1, 2$. Since demand and price are fixed, revenue is unaffected by the ordering decisions (we consider the case where price is a decision variable that affects demand in Section 3.5). First, the buyer chooses its order quantity; then does the supplier, taking into account the ordering decision of the buyer. It is not difficult to show that in such a setting the optimal ordering policy is a *nested* policy with the supplier ordering an integer multiple m_2 of the buyer's order quantity q_1 (see for example Chapter 2 of Zipkin (2000)). Note that the buyer is unaffected by the decisions of the supplier but the reverse is not true.

The problem faced by the buyer can be formulated as follows:

$$\max_{q_1}\ z_1(q_1) = (p - w - c_1)D - \frac{A_1 D}{q_1} - \frac{h_1 q_1}{2} - t(\frac{\hat{A}_1 D}{q_1} + \frac{\hat{h}_1 q_1}{2} + \hat{c}_1 D), \qquad (3.1)$$

while the problem faced by the supplier, given that the buyer orders quantity q_1, can be formulated as

$$\begin{aligned} \max_{m_2}\ z_2(m_2, q_1) &= (w - c_2)D - \frac{A_2 D}{m_2 q_1} - \frac{h_2(m_2 - 1)q_1}{2} \qquad (3.2) \\ &\quad -t(\frac{\hat{A}_2 D}{m_2 q_1} + \frac{\hat{h}_2(m_2 - 1)q_1}{2} + \hat{c}_2 D), \end{aligned}$$

where the second argument of the function z_2 is used to highlight the dependency of the profit function of the supplier on the decision of the buyer.

We let q_1^* denote the buyer's optimal order quantity and m_2^* the supplier's optimal order quantity multiplier. Also, let $\tilde{A}_i = A_i + t\hat{A}_i$ and $\tilde{h}_i = h_i + t\hat{h}_i$, for $i = 1, 2$. Then, we can show that

$$q_1^* = \sqrt{\frac{2\tilde{A}_1 D}{\tilde{h}_1}}, \qquad (3.3)$$

and m_2^* is an integer that satisfies the inequalities

$$m_2^*(m_2^*-1) \leq \frac{\tilde{A}_2\tilde{h}_1}{\tilde{A}_1\tilde{h}_2} \leq m_2^*(m_2^*+1). \tag{3.4}$$

Given order quantity q_1^* and order multiplier m_2^*, the buyer's and the supplier's emissions are respectively given by

$$e_1(q_1^*) = \frac{\hat{A}_1 D}{q_1^*} + \frac{\hat{h}_1 q_1^*}{2} + \hat{c}_1 D, \tag{3.5}$$

and

$$e_2(m_2^*, q_1^*) = \frac{\hat{A}_2 D}{m_2^* q_1^*} + \frac{\hat{h}_2(m_2^*-1)q_1^*}{2} + \hat{c}_2 D. \tag{3.6}$$

Total supply chain profit and total supply chain emission can then be obtained respectively as

$$z_{SC}(m_2^*, q_1^*) = z_1(q_1^*) + z_2(m_2^*, q_1^*),$$

and

$$e_{SC}(m_2^*, q_1^*) = e_1(q_1^*) + e_2(m_2^*, q_1^*).$$

Let us first consider the case where the supplier uses a *lot-for-lot* policy (i.e., $m_2 = 1$). By virtue of inequalities (3.4), the lot-for-lot policy is optimal when $0 \leq \frac{\tilde{A}_2\tilde{h}_1}{\tilde{A}_1\tilde{h}_2} \leq 2$. The lot-for-lot policy is commonly used even when it is not optimal because of its simplicity and because it is consistent with the principle of just-in-time adopted by many firms; see for example Corbett and De Groote (2000). The results below apply regardless of whether the lot-for-lot policy is optimal or not.

Under a lot-for-lot policy, the expression of supplier's emissions simplifies to

$$e_2(q_1^*) = \hat{A}_2\sqrt{\frac{\tilde{h}_1 D}{2\tilde{A}_1}} + \hat{c}_2 D, \tag{3.7}$$

and that of total supply chain emissions to

$$e_{SC}(q_1^*) = (\hat{A}_1 + \hat{A}_2)\sqrt{\frac{\tilde{h}_1 D}{2\tilde{A}_1}} + \hat{h}_1\sqrt{\frac{\tilde{A}_1 D}{2\tilde{h}_1}} + (\hat{c}_1 + \hat{c}_2)D, \qquad (3.8)$$

where, for ease of notation, we have dropped the argument $m_2(= 1)$ from the notation of the emission functions.

In the following theorem, we characterize the effect of the emission penalty on total supply chain emission.

Theorem 3 *Total supply chain emission $e_{SC}(q_1^*)$ is increasing in the emission penalty t if and only if $\frac{\hat{A}_1}{\hat{h}_1} \leq \frac{A_1}{h_1}$ and $\hat{A}_2 \geq \frac{\hat{h}_1 A_1 - h_1 \hat{A}_1}{\tilde{h}_1}$.*

The result follows immediately from noting that $\frac{\mathrm{d}e_{SC}(q_1^*)}{\mathrm{d}t} \geq 0$ if and only if the condition specified in the theorem holds. The following are immediate corollaries.

Corollary 2 *A sufficient condition, independent of t, for total supply chain emission to increase in the emission penalty is given by $\frac{\hat{A}_1}{\hat{h}_1} \leq \frac{A_1}{h_1}$ and $\hat{A}_2 \geq \frac{\hat{h}_1 A_1 - h_1 \hat{A}_1}{h_1}$.*

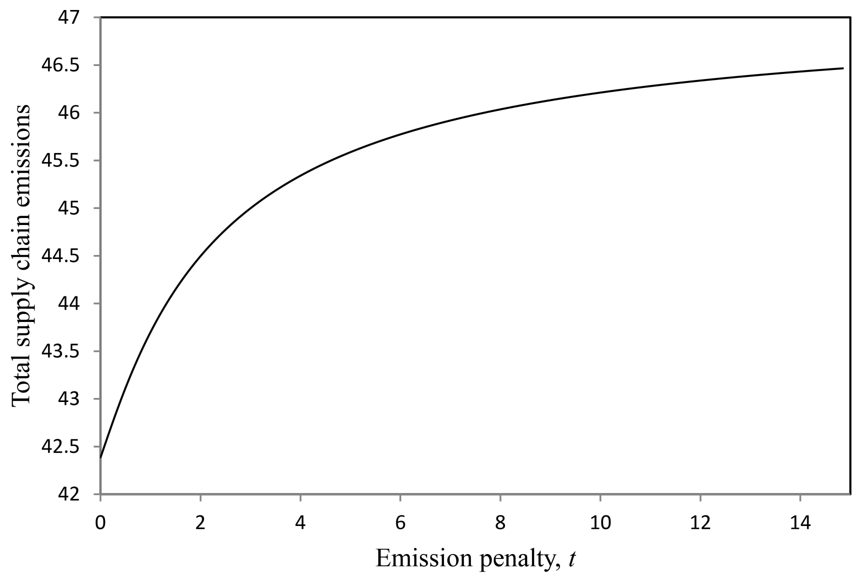
Corollary 3 *If $\frac{\hat{A}_1}{\hat{h}_1} \leq \frac{A_1}{h_1}$ but $\hat{A}_2 < \frac{\hat{h}_1 A_1 - h_1 \hat{A}_1}{\tilde{h}_1}$, then there exists a threshold t' on the emission penalty such that if $t \leq t'$, then total supply chain emission is decreasing in t and if $t > t'$, then supply chain emission is increasing in t, with $t' = \frac{\hat{h}_1 A_1 - h_1 \hat{A}_2 - h_1 \hat{A}_1}{\hat{h}_1 \hat{A}_2}$.*

The fact that higher emission penalties can lead to higher supply chain emission can be explained as follows. When $\frac{\hat{A}_1}{\hat{h}_1} \leq \frac{A_1}{h_1}$, the buyer's optimal order quantity q_1^* decreases with the emission penalty (the introduction of a penalty moves the buyer from the *cost-optimal* order quantity, given by $\sqrt{2A_1 D/h_1}$, in the direction of the *emission-optimal* order quantity, given by $\sqrt{2\hat{A}_1 D/\hat{h}_1}$). In doing so, the buyer of course reduces its own emissions. However, this comes at the expense of the supplier whose emissions increase. This is because, under lot-for-lot, the supplier uses the same order quantity as the buyer. Doing so leads the supplier to incur higher fixed emissions (associated with more frequent ordering) without benefiting from lower inventory-related emissions, as does the buyer (recall that, under lot-for-lot, the supplier does not keep any inventory). If the emission increase by the the supplier is higher than the emission reduction experienced by the buyer, then total supply chain emissions see a net increase. A condition for this to occur is for the supplier's emission parameter $\hat{A}_2$ to be larger than a threshold as specified by the condition in the theorem. Note that because this threshold is decreasing in t, it is possible for total emissions to decrease as t increases initially and then to increase as t increases further.

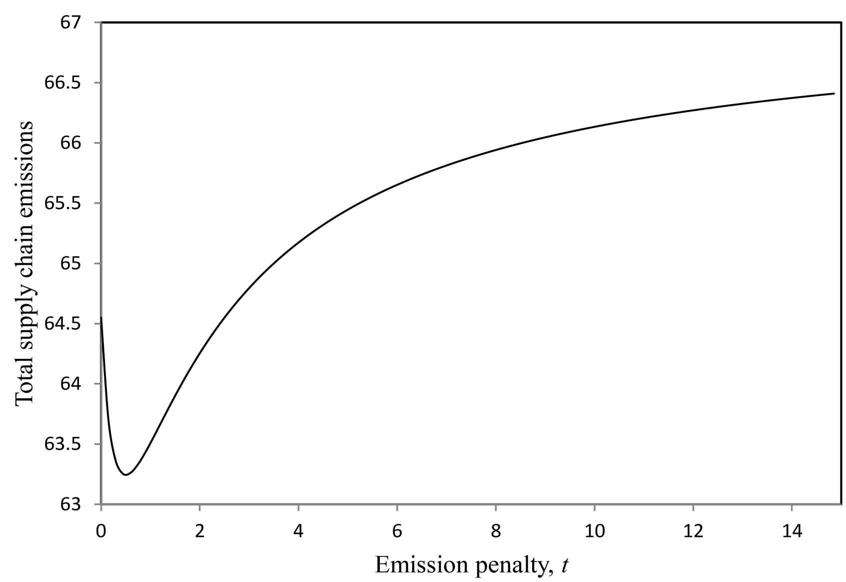
In figures 3.1(a) and 3.1(b), we illustrate the above results. Figure 3.1(a) shows that the increase in emissions can be significant (as large as 10% in the example shown). Figure 3.1(b), illustrates the non-monotonicity of emissions in the emission penalty t.

Higher emission penalties can also lead to higher total supply chain emission when the supplier does not use a lot-for-lot policy and instead uses a multiplier, other than one, of the buyer's order quantity (regardless of whether or not this multiplier is optimized). First, consider the case where the supplier uses an exogenously specified multiplier m_2 ($m_2 \geq 1$). Let $A'_2 = \frac{A_2}{m_2}$, $h'_2 = (m_2 - 1)h_2$, $\hat{A}'_2 = \frac{\hat{A}_2}{m_2}$ and $\hat{h}'_2 = (m_2 - 1)\hat{h}_2$. Then the total supply chain cost and emission can be rewritten, respectively, as follows:

$$z_{SC}(q_1^*, m_2) = \frac{(A_1 + A'_2)D}{q_1^*} + \frac{(h_1 + h'_2)q_1^*}{2} + (c_1 + c_2)D, \tag{3.9}$$



(a) $(A_2 = 10, h_2 = 1, \hat{A}_2 = 5, \hat{h}_2 = 1, A_1 = 6, h_1 = 1.1, \hat{A}_1 = 3, \hat{h}_1 = 1.1, \hat{c}_1 = \hat{c}_2 = 0, D = 100)$



(b) $(A_2 = 15, h_2 = 1.4, \hat{A}_2 = 10, \hat{h}_2 = 1.1, A_1 = 21, h_1 = 1, \hat{A}_1 = 5, \hat{h}_1 = 1, \hat{c}_1 = \hat{c}_2 = 0, D = 100)$

Figure 3.1: The impact of the emission penalty on supply chain emission

and

$$e_{SC}(q_1^*, m_2) = \frac{(\hat{A}_1 + \hat{A}_2')D}{q_1^*} + \frac{(\hat{h}_1 + \hat{h}_2')q_1^*}{2} + (\hat{c}_1 + \hat{c}_2)D. \tag{3.10}$$

Note that when $m_2 > 1$, the supplier holds inventory and therefore incurs inventory holding costs and emissions.

The following theorem provides necessary and sufficient conditions for the total emission to increase with the emission penalty.

Theorem 4 *Given multiplier m_2, total supply chain emission increases with the emission penalty t if and only if one of the following conditions holds:*

- $\frac{\hat{A}_1}{\hat{h}_1} \le \frac{A_1}{h_1}$ *and* $\hat{A}_2' \ge \frac{\tilde{A}_1(\hat{h}_1+\hat{h}_2')}{\tilde{h}_1} - \hat{A}_1$*, or*
- $\frac{\hat{A}_1}{\hat{h}_1} \ge \frac{A_1}{h_1}$ *and* $\hat{A}_2' \le \frac{\tilde{A}_1(\hat{h}_1+\hat{h}_2')}{\tilde{h}_1} - \hat{A}_1$.

The result again follows from verifying that $\frac{\mathrm{d}e_{SC}(q_1^*, m_2)}{\mathrm{d}t} \ge 0$ if and only if the conditions specified in the theorem hold. The result generalizes that of Theorem 1 and can be explained as follows. When $\frac{\hat{A}_1}{\hat{h}_1} \le \frac{A_1}{h_1}$, the buyer's optimal order quantity q_1^* decreases with the emission penalty, per the explanation given for the result in Theorem 3. This has again the effect of reducing the buyer's emission but increasing the supplier's fixed emission. When $\hat{A}_2' \ge \frac{\tilde{A}_1(\hat{h}_1+\hat{h}_2')}{\tilde{h}_1} - \hat{A}_1$, the increase in the supplier's fixed emissions is sufficiently large to lead to a net increase in total supply chain emission. When $\frac{\hat{A}_1}{\hat{h}_1} \ge \frac{A_1}{h_1}$, the buyer reduces its emission by increasing its optimal order quantity q_1^*. However, this has the effect of increasing the inventory-related emission of the supplier. if $\hat{A}_2' \le \frac{\tilde{A}_1(\hat{h}_1+\hat{h}_2')}{\tilde{h}_1} - \hat{A}_1$, the increase in the supplier's inventory-related emission is sufficiently large to lead to an increase in total supply chain emission. Note that when $m_2 = 1$ (the lot-for-lot case), only the first condition of Theorem 4 can hold. In that case, substituting for $m_2 = 1$ leads to the result in Theorem 3.

When m_2 is not fixed and is instead chosen optimally by the supplier, the optimal value m_2^* varies with the penalty rate t. However, for any value of m_2^*, there is always a non-empty interval for t, say $[t_1(m_2^*), t_2(m_2^*)]$, over which m_2^* is constant (recall that m_2 is an integer). For each such interval whose internal is not empty, and corresponding fixed m_2^*, it is possible for total supply chain emission to increase if one of the conditions in Theorem 4 are satisfied. In Figure 3.2, we illustrate this effect for an example system. Note that the discontinuities in total supply chain emission are due to the integrality of m_2.

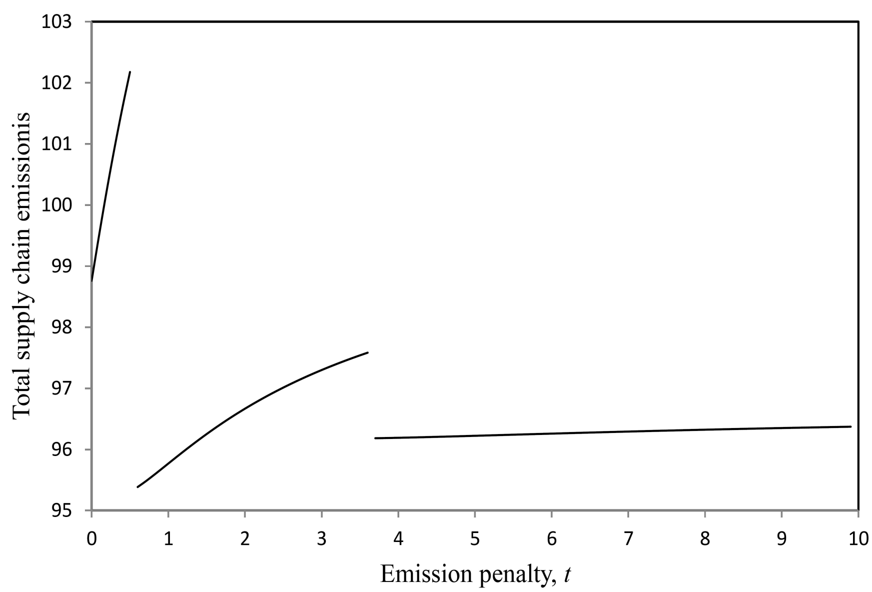


Figure 3.2: The impact of the emission penalty on supply chain emission when $m_2^* > 1$ ($A_2 = 10, h_2 = 1, \hat{A}_2 = 25, \hat{h}_2 = 1, A_1 = 5, h_1 = 1.5, \hat{A}_1 = 3, \hat{h}_1 = 2, \hat{c}_1 = \hat{c}_2 = 0, D = 100$)

We conclude this section by making the following two remarks. First, our analysis can be readily extended to serial supply chains with multiple firms, where each firm, other than the end buyer, is a supplier to the one directly downstream from it. We can show that, if the the conditions in Theorem 4 hold, the potential negative impact of higher emission penalties continues to be present and in fact can be amplified. For example, in the case where all suppliers are symmetric in their parameters and all employ a lot-for-lot policy, total supply chain emission increases linearly in the number of firms if the conditions of Theorem 3 hold.

Second, although we have considered settings where firms are subject to the same emission penalty, the potential negative impact of higher emission penalties is also present when the penalties faced by the firms are different or when only the buyer faces a penalty (e.g., this is easy to verify in the case of lot-for-lot policy). More significantly, this effect is present when neither firm is subject to an emission penalty but instead the buyer decides to voluntarily reduce its own emission down to a specified target (i.e., the buyer maximizes its profit subject to a constraint on emissions).

In Section 3.6, we discuss other examples of applications where a penalty on the emissions (or the emission reduction effort) of one or more of the firms can backfire and lead to higher overall emissions. We also a describe a general model for supply chain interactions, of which the various applications we discuss, including the model in this section, can be viewed as special cases.

3.4 Potential Remedies

In this section, we consider remedies that can mitigate the possibility of emission penalties backfiring and resulting in higher supply chain emission and not less.

A remedy would naturally require that the buyer internalizes either partially or fully the cost of the externality it imposes on the supplier. We consider three such remedies. The first involves the firms *coordinating* their ordering decisions, either explicitly or implicitly, so that total (after-penalty) supply chain profit is maximized (i.e., the buyer and the supplier adopt the *centralized* solution). The second is one where both the buyer and the supplier are penalized for a fraction of total supply chain emission instead of only the emissions for which they are directly responsible. The third involves penalizing consumption instead of production. This remedy applies to settings where demand is sensitive to pricing and we postpone its discussion until Section 3.5 where we consider the case of endogenous demand.

3.4.1 Supply Chain Coordination

It may be possible for the buyer and the supplier to coordinate their ordering decisions so that total supply chain profit is maximized. This could take place voluntarily, with the buyer and the supplier agreeing to collaborate and share the resulting additional profit. It could also be imposed by headquarters if the buyer and the supplier are business units within the same firm. Coordination could also occur if either the buyer or the supplier has enough market power or if either is in the position to enforce a coordinating contract (see for example Weng (1995), Corbett and De Groote (2000), Sarmah et al. (2006)) for a discussion of coordinating contracts. When the buyer and the supplier coordinate their ordering decisions so as to maximize total supply chain profit, the problem for the central planner can be formulated as follows:

$$\max_{m_2, q_1} \; z_{SC}(m_2, q_1).$$

The resulting optimal order quantity for the buyer is then given by

$$q_1^C = \sqrt{\frac{2(\frac{\tilde{A}_2}{m_2^C} + \tilde{A}_1)D}{\tilde{h}_1 + (m_2^C - 1)\tilde{h}_2}}, \tag{3.11}$$

while the optimal order quantity multiplier for the supplier is an integer that satisfies the inequalities

$$m_2^C(m_2^C - 1) \leq \frac{\tilde{A}_2(\tilde{h}_1 - \tilde{h}_2)}{\tilde{A}_1 \tilde{h}_2} \leq m_2^C(m_2^C + 1). \tag{3.12}$$

In the case when a lot-for-lot policy is used (optimal when $0 \leq \frac{\tilde{A}_2(\tilde{h}_1 - \tilde{h}_2)}{\tilde{A}_1 \tilde{h}_2} \leq 2$), the order quantity simplifies to

$$q_1^C = \sqrt{\frac{2(\tilde{A}_1 + \tilde{A}_2)D}{\tilde{h}_1}}, \tag{3.13}$$

and total supply chain emission is given by

$$e_{SC}(q_1^C) = (\hat{A}_1 + \hat{A}_2)\sqrt{\frac{\tilde{h}_1 D}{2(\tilde{A}_1 + \tilde{A}_2)}} + \hat{h}_1 \sqrt{\frac{(\tilde{A}_1 + \tilde{A}_2)D}{2\tilde{h}_1}} + (\hat{c}_1 + \hat{c}_2)D. \tag{3.14}$$

It is easy to see that the order quantity for the buyer is always larger than the order quantity in the decentralized system (the optimal order quantity now accounts for the fixed ordering cost at both the buyer and the supplier). It is obvious that with coordination a higher emission penalty always leads to lower total supply chain emission and it can be directly verified that $\frac{\mathrm{d}e_{SC}(q_1^C)}{\mathrm{d}t} < 0$. Hence, if feasible, supply chain coordination would always ensure that imposing a penalty on emissions (or increasing the penalty) always has the intended consequence of reducing emissions.

However, we must caution that coordination may not always be feasible or practical. For example, decentralized supply chains are prevalent in practice and so are contracts that do not guarantee coordination such as those involving wholesale prices. More importantly, we must caution that coordination may not necessarily lead to emissions that are lower than those experienced in the absence of coordination. In fact, as we show below, given the same emission penalty, it is possible for a coordinated supply chain to have higher emissions than a decentralized one. To see this, consider the case of a lot-for-lot policy (the result readily extends to the more general case). Then the following holds.

Proposition 6 *Given a penalty t, a coordinated supply chain has strictly higher emissions than a non-coordinated supply chain if and only if* $\frac{\hat{h}_1}{\tilde{h}_1} > \frac{\hat{A}_1+\hat{A}_2}{\sqrt{(\tilde{A}_2+\tilde{A}_2)\tilde{A}_2}}$.

The proof follows immediately from comparing total supply chain emission when there is coordination $(e_{SC}(q_1^C))$with supply chain emission in the absence of coordination $(e_{SC}(q_1^*))$.

The result in Proposition 6 can be explained as follows. With coordination, the order quantity increases (relative to the order quantity without coordination). This leads to lower fixed emissions for both the buyer and the supplier but higher inventory-related emissions for the buyer (the supplier does not carry any inventory). If the decrease in the fixed emissions is less than the increase in the inventory-related emissions, total supply chain emission would see a net increase. This can be most readily seen in the special case of $t = 0$, where the condition in Proposition 6 can be rewritten as $\hat{h}_1 > \frac{h_1(\hat{A}_1+\hat{A}_2)}{\sqrt{(A_1+A_2)A_2}}$. In other words, if the buyer's inventory emission intensity is sufficiently high, then total supply chain emission in the coordinated supply chain would be higher. In all cases, coordination of course leads to higher supply chain profit. However, with coordination, firms may choose to reduce operational cost at the expense of increasing emission cost. Figure 3.3 illustrates for an example system with and without coordination and, as we can see, a coordinated supply chain could have a significantly higher emission.

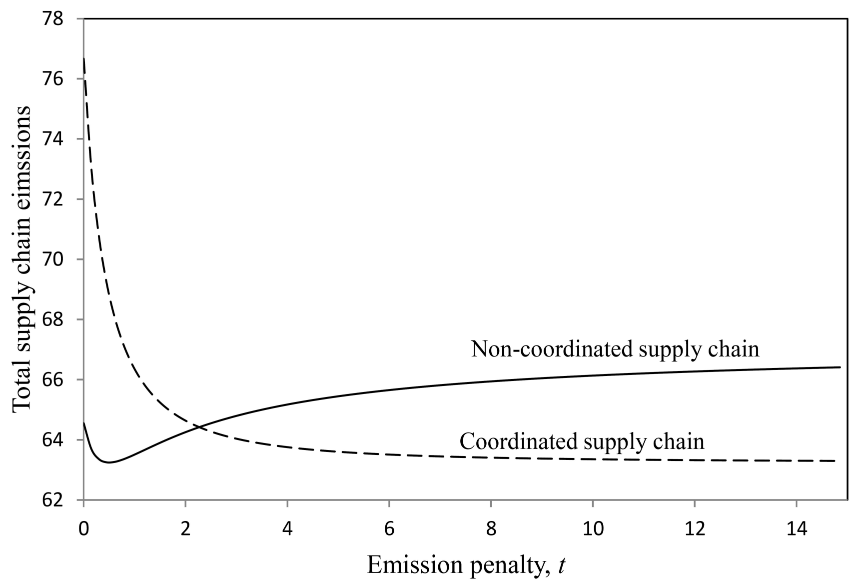


Figure 3.3: Supply chain emission with and without coordination ($A_2 = 15, h_2 = 1.4, \hat{A}_2 = 10, \hat{h}_2 = 1.1, A_1 = 21, h_1 = 1, \hat{A}_1 = 5, \hat{h}_1 = 1, \hat{c}_1 = \hat{c}_2 = 0, D = 100$)

3.4.2 Shared Emission Responsibility

An alternative to penalizing each firm for its own emission is to penalize each firm for a fraction of total supply chain emission. We refer to such a scheme as *shared emission responsibility.* Shared responsibility clearly ensures that emission externalities inflicted by one firm on another are internalized. Shared emission responsibility could be imposed if the firms correspond to business units within the same firm or if the firms voluntarily agree to share the emission burden. The latter might arise if firms are held accountable for the emissions of their entire supply chain. For example, a growing number of publicly-traded firms are being pressured by their shareholders to disclose not only their own emissions but the emissions of their suppliers and customers; see CDP (2013b). It might also arise if firms see an opportunity in positioning their products as having an overall low emission footprint.

Under shared emission responsibility, firm i, for $i = 1, 2$, is penalized for a fraction $\alpha_i \geq 0$ of total supply chain emission, with $\sum_{i=1}^{2} \alpha_i = 1$. Then, given penalty t and emission allocation ($\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$), the buyer's problem is given by (for simplicity, we assume the supplier follows a lot-for-lot policy, but the analysis can again be extended to the general case)

$$\begin{aligned} \max_{q_1} \; z_1(q_1) &= (p-w)D - (\frac{A_1 D}{q_1} + \frac{h_1 q_1}{2} + c_1 D) \\ &\quad -\alpha t(\frac{(\hat{A}_1 + \hat{A}_2)D}{q_1} + \frac{\hat{h}_1 q_1}{2} + (\hat{c}_1 + \hat{c}_2)D), \end{aligned} \tag{3.15}$$

with a corresponding optimal order quantity given by

$$q_1^*(\alpha) = \sqrt{\frac{2[A_1 + \alpha t(\hat{A}_1 + \hat{A}_2)]D}{h_1 + \alpha t \hat{h}_1}}. \tag{3.16}$$

Given $q_1^*(\alpha)$, the supplier's profit is given by

$$\begin{aligned} z_2(q_1^*(\alpha)) &= wD - (\frac{A_2 D}{q_1^*(\alpha)} + c_2 D) - (1-\alpha)t(\frac{(\hat{A}_1 + \hat{A}_2)D}{q_1^*(\alpha)} \\ &\quad + \frac{\hat{h}_1 q_1^*(\alpha)}{2} + (\hat{c}_1 + \hat{c}_2)D). \end{aligned} \tag{3.17}$$

From the above, it is easy to verify that supply chain emission is decreasing in t.

Although any allocation that satisfies $\sum_{i=1}^{2} \alpha_i = 1$ and $\alpha_i \geq 0$ accounts for all the emission and guarantees that penalties lead to lower total supply chain emission, different allocations lead to different costs and to different levels of emissions for both the individual firms and the supply chain. In the remainder of this section, we examine the impact of three plausible allocation mechanisms. The first allocates emission responsibility so that total supply chain profit is maximized; the second so that total emission is minimized; and the third so that emission responsibility is proportional to the base-line emissions in the absence of penalties.

An allocation that maximizes supply chain profit is obtained by choosing α such that $z_{SC}(q_1^*(\alpha))$ is maximized. The implementation of this allocation could be the outcome of a negotiation between the buyer and the supplier, which may include additional financial transfers between the two parties, or imposed by a third a party, such as headquarters in the case where the buyer and the supplier are business units within the same firm. The optimal emission allocation problem can be stated as follows:

$$\begin{aligned} \max_{\alpha}\ z_{SC}(q_1^*(\alpha)) &= (p-w)D - (\frac{(A_1+A_2)D}{q_1^*(\alpha)} + \frac{h_1 q_1^*(\alpha)}{2} \\ &\quad +(c_1+c_2)D) - t(\frac{(\hat{A}_1+\hat{A}_2)D}{q_1^*(\alpha)} + \frac{\hat{h}_1 q_1^*(\alpha)}{2} + (\hat{c}_1+\hat{c}_2)D), \\ \text{subject to} &\quad \sum_{i=1}^{2} \alpha_i = 1 \text{ and } \alpha_i \geq 0. \end{aligned} \tag{3.18}$$

If we ignore the constraints on α, we can show that the optimal allocation is given by

$$\alpha^* = \frac{\frac{A_2}{\hat{h}_1} + t(\frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} - \frac{A_1}{h_1})}{t(\frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} - \frac{A_1+A_2}{h_1})},$$

and the corresponding optimal order quantity is

$$q_1^*(\alpha^*) = \sqrt{\frac{2[A_1 + A_2 + t(\hat{A}_1 + \hat{A}_2)]D}{h_1 + t\hat{h}_1}}.$$

This order quantity is the same as the optimal order quantity in the coordinated system since optimizing with respect to α is equivalent to optimizing with respect to q_1. This means that, whenever it is feasible, shared emission responsibility with an unconstrained allocation can coordinate the supply chain. However, the unconstrained optimal allocation may not be feasible if either $\alpha^* > 1$ or $\alpha^* < 0$. The former occurs if $\frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} \geq \frac{A_1+A_2}{h_1}$ and the latter occurs if $\frac{A_1}{h_1} - \frac{A_2}{t\hat{h}_1} < \frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} < \frac{A_1+A_2}{h_1}$. Given the unimodularity of the function z_{SC} with respect to α, it is optimal to set α^* to 1 in the case of the former and 0 in the case of the latter. The solution to the constrained problem in (3.18) is summarized below.

Proposition 7

$$\alpha^* = \begin{cases} 0 & \textit{if } \frac{A_1}{h_1} - \frac{A_2}{t\hat{h}_1} < \frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} < \frac{A_1+A_2}{h_1}, \\ 1 & \textit{if } \frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} > \frac{A_1+A_2}{h_1}, \textit{ and} \\ \frac{\frac{A_2}{\hat{h}_1} + t(\frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} - \frac{A_1}{h_1})}{t(\frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} - \frac{A_1+A_2}{h_1})} & \textit{otherwise,} \end{cases} \tag{3.19}$$

and

$$q_1^*(\alpha^*) = \begin{cases} \sqrt{\frac{2A_1D}{h_1}} & \textit{if } \frac{A_1}{h_1} - \frac{A_2}{t\hat{h}_1} < \frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} < \frac{A_1+A_2}{h_1}, \\ \sqrt{\frac{2[A_1+t(\hat{A}_1+\hat{A}_2)]D}{h_1+t\hat{h}_1}} & \textit{if } \frac{\hat{A}_1+\hat{A}_2}{\hat{h}_1} > \frac{A_1+A_2}{h_1}, \textit{ and} \\ \sqrt{\frac{2[A_1+A_2+t(\hat{A}_1+\hat{A}_2)]D}{h_1+t\hat{h}_1}} & \textit{otherwise.} \end{cases}$$

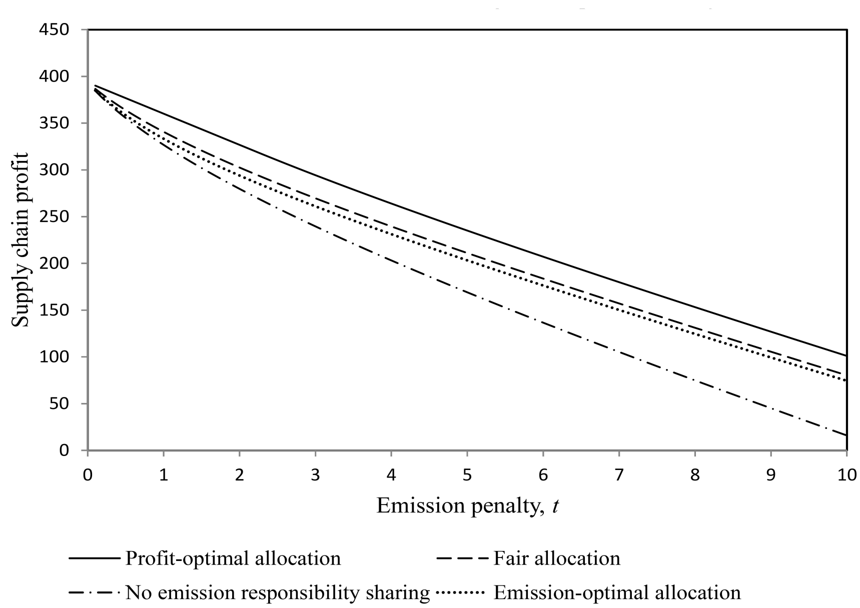
The second allocation we consider is one that, instead of maximizing supply chain profit, minimizes total supply chain emission. In this case, it is straightforward to show that it is optimal to set $\alpha = 1$ and the optimal order quantity to $\sqrt{\frac{2[A_1+t(\hat{A}_1+\hat{A}_2)]D}{h_1+t\hat{h}_1}}$. That is, in order to ensure that total emission in the supply chain is minimized, we must allocate the entire emission responsibility to the buyer.

Both the *profit-optimal* and *emission-optimal* allocations require that the firms share their cost and emission parameters with each other or with a neutral third party verifier. Also, both may be perceived as *unfair*, given that one firm could be assigned most or all of the emission responsibility. To address these concerns, the third allocation we consider is one that assigns emission responsibility proportionally to the business-as-usual emissions of each firm. That is, a firm is responsible for a fraction of total supply chain emission equal to the fraction of total emission for which it would have been responsible absent the emission penalty. We let α^F denote the emission fraction assigned to the buyer. Then, α^F is specified as follows:

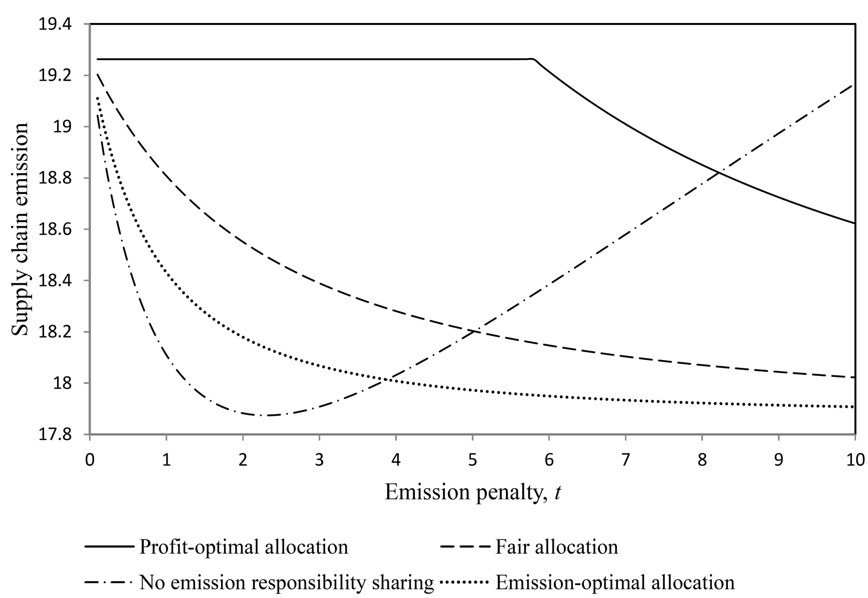
$$\alpha^F = \frac{\frac{\hat{A}_1 D}{q^*_{1,BAU}} + \frac{\hat{h}_1 q^*_{1,BAU}}{2} + \hat{c}_1 D}{\frac{(\hat{A}_1+\hat{A}_2)D}{q^*_{1,BAU}} + \frac{\hat{h}_1 q^*_{1,BAU}}{2} + (\hat{c}_1 + \hat{c}_2)D},$$

where $q^*_{1,BAU} = \sqrt{\frac{2A_1 D}{h_1}}$ and corresponds to the order quantity chosen by the buyer under business-as-usual (BAU). This allocation is arguably fairer than the other two and requires no information sharing (beyond disclosing business-as-usual emissions).

In Figures 3.4(a) and 3.4(b), we illustrate for an example system the performance of the three different allocations, along with the performance of the system where no sharing takes place and each firm takes full responsibility for its own emission (i.e, the system without coordination). As we can see, the different



(a) $D = 100, p = 4.5, A_1 = 1, h_1 = 1, A_2 = 6, c_1 = 0, c_2 = 0, \hat{A}_1 = 0.1, \hat{h}_1 = 3, \hat{A}_2 = 0.2, \hat{c}_1 = 0, \hat{c}_2 = 0.1$



(b) $D = 100, A_1 = 1, h_1 = 1, A_2 = 4, c_1 = 0, c_2 = 0, \hat{A}_1 = 0.01, \hat{h}_1 = 1, \hat{A}_2 = 0.3, \hat{c}_1 = 0, \hat{c}_2 = 0.1$

Figure 3.4: Supply chain profit and emission under different penalty schemes

approaches can lead to different profit and emission profiles. Although the profit-optimal allocation achieves the highest profit, it can lead to significantly higher emissions (up to 6.1% higher than the *fair* allocation and 7.3% higher than the *emission-optimal* allocation in the example shown). Similarly, the fair allocation and the emission-optimal allocation can lead to significantly lower profit than the profit-optimal allocation (up to 20.5% for the fair allocation and up to 26.3% for the emission-optimal allocation). More significantly, all three shared emission responsibility schemes can lead to higher emissions than those incurred in the absence of emission responsibility sharing.

3.5 Systems with Endogenous Demand

In this section, we extend our analysis to the case where the price of the end product is a decision made by the buyer, with demand being sensitive to price (a higher price implies lower demand) and show that it is still possible for emission penalties to backfire. To allow for closed form results, we consider the case where demand follows the widely studied constant elasticity demand function $D(p) = dp^{-2}$, where $d > 0$, but the analysis can be carried out for more general demand functions. Also, for ease of exposition, we assume that the supplier follows a lot-for-lot policy.

Assuming that, for a given price p, the buyer chooses order quantity $q_1(p) = \sqrt{\frac{2\tilde{A}_1 D(p)}{\tilde{h}_1}}$, the buyer's problem can be stated as follows (see Weng (1995) for a similar model and analysis):

$$\max_p \; z_1(p) = (p - w - \tilde{c}_1)D(p) - \sqrt{2\tilde{A}_1\tilde{h}_1 D(p)}. \tag{3.20}$$

Given the buyer's optimal price p^*, the supplier's profit is given by

$$z_2(p^*) = (w - \tilde{c}_2)D(p^*) - \tilde{A}_2\sqrt{\frac{\tilde{h}_1 D(p^*)}{2\tilde{A}_1}}.$$

Substituting dp^{-2} for $D(p)$, it is easy to verify that the profit function of the buyer is unimodal in price, with the optimal price given by

$$p^* = \frac{2(w + \tilde{c}_1)}{1 - \sqrt{2\tilde{A}_1\tilde{h}_1/d}}, \tag{3.21}$$

and corresponding optimal demand $D(p^*) = dp^{*-2}$.

From (3.21) we can verify that p^* is increasing in t, resulting in demand that is decreasing in t. Consequently, emissions that scale up with demand also decrease with t. We refer to this effect as the demand *curtailment* effect. However, the emission reduction due to demand curtailment may not be sufficient to result in a net reduction in total supply chain emission. As we show in the theorem below, it may still be possible for higher emission penalties to result in higher supply chain emissions.

Theorem 5 *Total supply chain emission increases with the emission penalty t if and only the following condition holds:*

$$\frac{A_1}{h_1} \geq \frac{\hat{A}_1}{\hat{h}_1}, \hat{A}_2 \geq \hat{A}_2^L, \ \ \textit{and} \ \ -\frac{\frac{\mathrm{d}D(p^*)}{\mathrm{d}t}}{D(p^*)} \leq \frac{A_1\hat{h}_1 - \hat{A}_1 h_1}{\tilde{A}_1\tilde{h}_1}.$$

where $\hat{A}_2^L = \frac{A_1\hat{h}_1 - \hat{A}_1 h_1}{\tilde{h}_1} - \frac{(\frac{\hat{h}_1}{2D(p^*)}q_1(p^*) + \hat{c}_1 + \hat{c}_2)\frac{\mathrm{d}D(p^*)}{\mathrm{d}t}}{\frac{\mathrm{d}}{\mathrm{d}t}(\frac{D(p^*)}{q_1(p^*)})}.$

To understand the conditions in Theorem 5, note that the first two are similar to those of Theorem 3 for the exogenous demand case. In particular, the second condition is a requirement that the fixed emission parameter $\hat{A}_2$ is sufficiently large (in fact, the requirement on $\hat{A}_2$ in Theorem 5 implies the requirement on $\hat{A}_2$ in Theorem 3, $\hat{A}_2 \geq \frac{A_1\hat{h}_1 - \hat{A}_1 h_1}{\hat{h}_1}$). The third inequality guarantees that the demand curtailment (and the associated emission reduction) is sufficiently small. That is, despite the emission penalty, demand continues to be sufficiently large. This requirement is specified in terms of a threshold on the relative decrease in demand as t increases. Note that this third condition is explicit and can be written in closed form.

With regard to remedies, it is easy to show that those considered for the case of exogenous demand continue to be effective. Given that price is now a decision variable (and demand is sensitive to price), we can in addition consider remedies that involve penalizing the consumer, either fully or partially, for supply chain emission. In particular, let us consider the case where the consumer, instead of the firms, is penalized for total supply chain emission. Such a scheme would of course require that emissions over the entire supply chain are documented and shared with the consumer.

Under a *consumption-based* penalty scheme, the consumer faces an effective price, p_e, that is the sum of the price the buyer charges and the emission penalty. This effective price, in turn, determines demand. Hence, the buyer's problem can be restated as one involving choosing effective price and order quantity. In other words, the buyer's problem is given by:

$$\begin{aligned} \max_{p_e} \; z_1(p_e) \;\; &= \;\; (p_e - t[\frac{\hat{A}_1 + \hat{A}_2}{q_1(p_e)} + \frac{\hat{h}_1 q_1(p_e)}{2D(p_e)} \\ &\quad +(\hat{c}_1 + \hat{c}_2)] - w - c_1)D(p_e) - \frac{A_1 D(p_e)}{q_1(p_e)} - \frac{h_1 q_1(p_e)}{2}, \end{aligned} \tag{3.22}$$

It is easy to see that the above problem is the same as the one the buyer faces when it is allocated all the emission cost (i.e., when $\alpha = 1$ under a shared emission responsibility scheme). In particular, both schemes lead the buyer to choose the same demand level and order quantity, leading to the same buyer and supplier profits, emission levels, and consumer surplus.

In Figure 3.5, we illustrate the differences in performance of five different penalty schemes involving different ways of allocating emission responsibility. To allow for a fair comparison between the different schemes, we evaluate the resulting *social welfare* for each, where social welfare is defined as: consumer surplus + producer surplus (supply chain profit) + revenue collected through the penalty scheme - environmental damage. Consumer surplus is defined in the usual way as $\int_{p^*}^{+\infty} D(p)\mathrm{d}p$, while environmental damage is given by $\omega e_{SC}(q_1(p^*), p^*) = \omega(\frac{(\hat{A}_1+\hat{A}_2)D(p^*)}{q_1(p^*)} + \frac{\hat{h}_1 q_1(p^*)}{2} + (\hat{c}_1 + \hat{c}_2)D(p^*)$, where ω corresponds to the social cost of one unit of emission (for the consumption-based penalty scheme, p^* is replaced by p_e^*).

The first of the penalty schemes depicted in Figure 3.5 corresponds to the one that allocates emission responsibility among the two firms so that total supply chain profit is maximized (profit-optimal emission allocation); the second corresponds to allocating responsibility proportionally to business-as-usual emissions (fair emission allocation); the third corresponds to allocating responsibility so that the social welfare is maximized (socially-optimal allocation); the fourth corresponds to the baseline scheme of penalizing each firm for its own emissions (no emission responsibility sharing); and the fifth corresponds to penalizing the consumer and not the firms (consumption-based emission penalty).

From the figure, we can make the following observations:

- As expected, the socially-optimal allocation dominates the other emission responsibility sharing schemes. More significantly, there are ranges of t for which these other allocations (the fair allocation, the profit-optimal allocation, and the consumption-based penalty) perform poorly with regard to social welfare.

- There are ranges of values of t for which the no emission responsibility sharing scheme outperforms all the other penalty schemes, implying that a policy that is socially-optimal may not necessarily involve fractional sharing of total supply chain emission. In fact, a scheme, where firm i is penalized at rate t_{ij} for emission due to firm j, with t_{ij} possibly different from t_{ji} can be shown to outperform all the schemes considered so far (and of which all can be viewed as special cases). Such a differentiated penalty scheme may, however, be difficult to implement when the price of emission is determined by a common market (e.g., a market for offsets or a market for tradable emission credits).

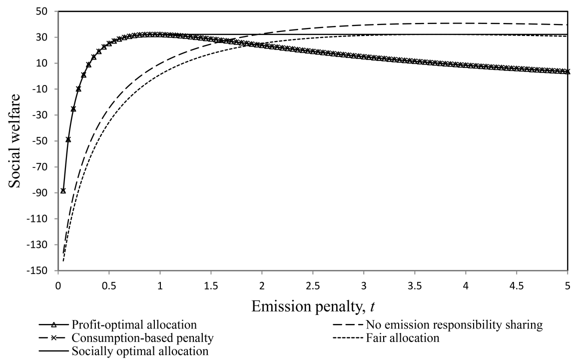
- There is a value of t that maximizes social welfare for each of the penalty schemes considered. Perhaps surprisingly, this value can be either lower or higher than the social cost of emission (ω). In some cases, a value lower than the social cost may be necessary so that consumer surplus is not degraded too much because of demand curtailment (this is consistent with results from environmental economics; see the discussion in Section 2). In other cases, a value higher than the social cost (i.e., overcompensating for emissions) may be preferable if a higher penalty reduces supply chain costs. For example, a higher penalty can lead the buyer to increase its order quantity which then reduces the supplier's fixed ordering cost. In turn, this can lead to an overall reduction in total supply chain cost.

3.6 Other Examples

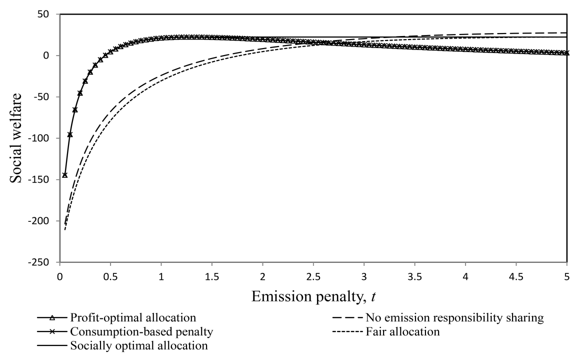
In this section, we show that higher penalties leading to higher supply chain emission is not unique to the buyer-supplier problem we have studied so far and that it arises in other settings. In particular, we discuss two other example applications. The first involves make versus buy decisions, where a manufacturer decides on how much to produce in-house and how much to outsource. The second involves retail store density decisions, where a retailer decides on how many stores to operate in a region. We also describe a general model formulation, of which the applications we discuss can be viewed as special cases.

3.6.1 A Make versus Buy Problem

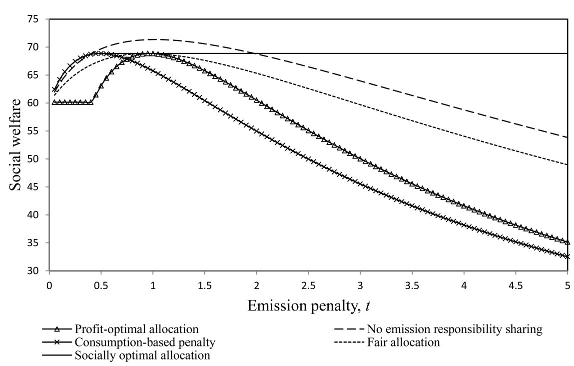
Consider two firms, a manufacturer and a supplier. The manufacturer decides on how much to produce in-house and how much to outsource to an external supplier. The supplier, in turn, decides on how much to charge the manufacturer. The manufacturer must fulfill a constant demand which, for simplicity, we normalize to one. The selling price, which we denote by p, is exogenously specified (the analysis can be extended to the case where the manufacturer also decides on the price). The manufacturer decides on the fraction of demand to produce in-house, which we denote by x_1, where $0 \leq x_1 \leq 1$, and the corresponding fraction $x_2 = 1 - x_1$ to outsource to the supplier (x_1 can be alternatively viewed as the amount of work content associated with each unit of the end product the manufacturer carries out



(a) $(d = 400, A_1 = 0.2, h_1 = 8, A_2 = 15, h_2 = 1.5, c_1 = 1, c_2 = 0.5, w = 1, \hat{A}_1 = 1, \hat{h}_1 = 0.5, \hat{A}_2 = 2, \hat{h}_2 = 2, \hat{c}_1 = 0, \hat{c}_2 = 1, \omega = 0.1)$



(b) $(d = 400, A_1 = 0.2, h_1 = 8, A_2 = 15, h_2 = 1.5, c_1 = 1, c_2 = 0.5, w = 1, \hat{A}_1 = 1, \hat{h}_1 = 0.5, \hat{A}_2 = 2, \hat{h}_2 = 2, \hat{c}_1 = 0, \hat{c}_2 = 1, \omega = 1)$



(c) $(d = 600, A_1 = 0.2, h_1 = 8, A_2 = 0, h_2 = 1.5, c_1 = 1, c_2 = 1, w = 2, \hat{A}_1 = 0.5, \hat{h}_1 = 0, \hat{A}_2 = 0, \hat{h}_2 = 0, \hat{c}_1 = 0.5, \hat{c}_2 = 1, \omega = 3)$

Figure 3.5: The impact of the emission penalty on social welfare

in-house, and $1 - x_1$ the work content that is outsourced). The supplier decides on the wholesale price w to charge the manufacturer.

The manufacturer and the supplier both face production cost and emission functions that are increasing and convex in the fraction of demand they are allocated. The convexity reflects settings where producing more becomes progressively more expensive and more emission-intensive. This would occur, for example, if limitations on capacity lead to more congestion (longer production leadtimes and higher levels of work-in-process inventories) or poorer quality and lower yields, all associated with higher cost and higher emissions and all typically convex in the amount of work in the system.

We model the interaction between the supplier and the manufacturer as a Stackelberg game. The manufacturer acts as the Stackelberg follower. It observes w and chooses the amount of demand (or work content) to fulfill in-house. The supplier, acts as the leader and chooses the price it charges the manufacturer, anticipating the decision of the manufacturer. For simplicity, we consider the case where the cost and emission functions are quadratic in the amount of demand (or work content) allocated to each firm. In particular, for specified x_1, the manufacturer incurs production cost $a_1x_1^2$ and emission $\hat{a}_1x_1^2$ while the supplier incurs production cost $a_2(1-x_1)^2$ and emission $\hat{a}_2(1-x_1)^2$, where $a_i, \hat{a}_i \geq 0$ for $i = 1, 2$.

Given price w, the manufacturer's problem can be stated as

$$\max_{x_1 \in [0,1]} z_1(x_1, w) = p - a_1x_1^2 - w(1 - x_1) - t\hat{a}_1x_1^2. \tag{3.23}$$

We can show that the manufacturer's best response function is given by

$$x_1^*(w) = \begin{cases} 1, & \text{if } w \geq 2\tilde{a}_1 \\ \frac{w}{2\tilde{a}_1}, & \text{otherwise} \end{cases},$$

where $\tilde{a}_1 = a_1 + t\hat{a}_1$.

The problem of the supplier, who anticipates the manufacturer's choice $x_1^*(w)$, is given by

$$\max_{w \geq 0} z_2(w, x_1^*(w)) = \quad w(1 - x_1^*(w)) - a_2(1 - x_1^*(w))^2 - t\hat{a}_2(1 - x_1^*(w))^2. \quad (3.24)$$

Using the fact that the relationship $w = 2\tilde{a}_1 x_1^*(w)$ always holds, the supplier's problem can be restated as follows:

$$\max_{x_1 \in [0,1]} 2\tilde{a}_1 x_1(1 - x_1) - \tilde{a}_2(1 - x_1)^2,$$

where $\tilde{a}_2 = a_2 + t\hat{a}_2$. The supplier's profit function is concave in x_1 and we can show that the optimal solution is given by

$$x_1^* \quad = \quad \frac{\tilde{a}_1 + \tilde{a}_2}{2\tilde{a}_1 + \tilde{a}_2}, \quad (3.25)$$

with a corresponding optimal wholesale price

$$w^* \quad = \quad \frac{2\tilde{a}_1(\tilde{a}_1 + \tilde{a}_2)}{2\tilde{a}_1 + \tilde{a}_2}. \quad (3.26)$$

The following proposition follows directly from the expressions in (3.25) and (3.26).

Proposition 8 *The optimal wholesale price of the supplier w^* is always increasing in t. However, the optimal amount produced in-house x_1^* is increasing in t if and only if $\frac{\hat{a}_1}{a_1} \leq \frac{\hat{a}_2}{a_2}$.*

It is interesting to note that, while higher emission penalties always lead to higher supplier prices (this is perhaps expected since the cost of both supplier and manufacturer are both higher), higher emission penalties may or may not lead to less outsourcing. The manufacturer outsources less if it has a lower emission to production cost ratio than that of the supplier and outsources more otherwise. This is because, in the case of the former higher emission penalties increase the relative cost advantage of producing in-house while in the latter, it improves that of outsourcing.

Theorem 6 *Total supply chain emission increases in the emission penalty if and only if one of the following conditions holds:*

- $\frac{\hat{a}_2}{a_2} \geq \frac{\hat{a}_1}{a_1}$ *and* $t \geq \frac{\hat{a}_2 a_1 - \hat{a}_1 a_1 - \hat{a}_1 a_2}{\hat{a}_1^2}$, *or*
- $\frac{\hat{a}_2}{a_2} < \frac{\hat{a}_1}{a_1}$ *and* $t \leq \frac{\hat{a}_2 a_1 - \hat{a}_1 a_1 - \hat{a}_1 a_2}{\hat{a}_1^2}$.

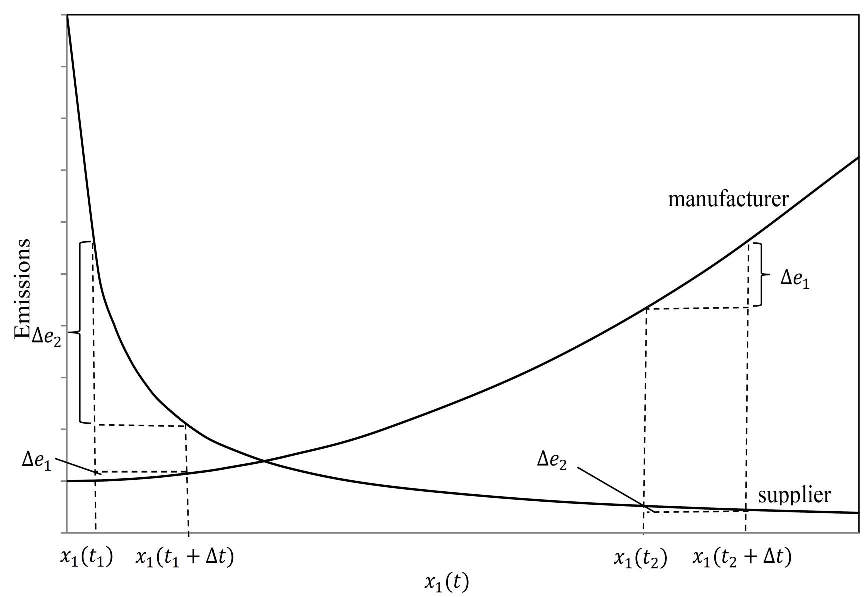


Figure 3.6: The impact of the emission penalty on the manufacturer's emission e_1 and the supplier' emissions e_2

The increase in supply chain emission can be explained as follows. Consider first the case when $\frac{\hat{a}_2}{a_2} \geq \frac{\hat{a}_1}{a_1}$. In this case, the manufacturer outsources more as the emission penalty increases (Proposition 8). The manufacturer's emission is convex increasing in the amount produced in-house (x_1). On the other hand the supplier's emission is convex decreasing in x_1. Hence, for sufficiently large t, or equivalently sufficiently large x_1, the rate of increase in the manufacturer's emission could exceed the rate of decrease in the supplier's emission, with the net effect being an increase in supply chain emission with an increase in t. This effect is illustrated graphically in Figure 3.6. The case of $\frac{\hat{a}_2}{a_2} < \frac{\hat{a}_1}{a_1}$ can be explained similarly, except that now an increase in the emission penalty leads to more outsourcing (Proposition 8). If t is sufficiently small, the increase in the supplier's emission rate exceeds the reduction in that of the manufacturer with the net effect being an increase in supply chain emission with an increase in t.

Potential remedies along the lines discussed in Section 3.4 can be constructed. Additionally, the analysis can be extended to the case where the manufacturer also decides on the selling price and demand is decreasing in this price (for brevity, we omit the details).

3.6.2 A Store Density Problem

Consider the problem posed recently by Cachon (2014) of a retailer who decides on the number and size of stores to serve a specified geographic region over which consumer are uniformly and continuously distributed. Consumers travel to the nearest store using the shortest path and along a straight line. Deliveries to the store are made from a warehouse that is co-located with one of the stores. Deliveries to the stores are carried out such that total delivery distances are minimized. In other words, the retailer solves a traveling salesman problem (TSP). The stores are located such that the region is segmented into equally sized sub-regions, with a store at the center of each subregion.

Consumers are more likely to shop with the retailer the shorter the distance they have to travel to a store. Hence more stores generate more demand for the retailer[3] . However, more stores imply longer delivery routes and, therefore, more transportation cost and possibly other fixed and variable costs for the retailer. Hence, the retailer makes a decision regarding the optimal number of stores by trading off the higher revenue generated from more stores with the corresponding higher cost.

We assume that sub-regions are similarly shaped polygons and, for ease of analysis, we restrict our treatment, to equilateral hexagons, squares, or triangles. We let a denote the area of the entire region, so that each sub-region has area a/n, where n denotes the number of stores. Given these assumptions, we can show that the average travel distance for the consumer is given by $d_c = \phi_c n^{-\frac{1}{2}}$ while the transportation distance for the retailer is given by $d_r = \phi_r n^{\frac{1}{2}}$, where ϕ_c and ϕ_r are constants that depend only on the shape of the sub-regions (here the subscripts c and r refer respectively to the consumer and the retailer); see Cachon (2014) for details and further discussion.

Both the retailer and consumers incur transportation costs and emissions per unit of demand per unit of distance traveled, which we denote by c_i and $\hat{c}_i$, for $i = r, c$ respectively. In addition the retailer incurs an emission penalty t per unit of emissions. These unit costs (emissions) may reflect fixed and variable costs (emissions) and account for the quantity transported by either the consumers

[3] Cachon (2014) treats the case where consumers are insensitive to travel distances and would always shop at the nearest store.

or the retailers[4] . It is possible to consider other costs, including fixed and variable costs associated with operating each store. We omit these for the sake of brevity, but the analysis can be easily extended to those cases and our main insight discussed below continues to hold (see also discussion at the end of this section).

We assume the overall demand function for the retailer, arising from the decisions of individual consumers, is decreasing in the average traveling distance (or equivalently, average traveling cost) of the consumers and given by $D = D_0(c_c d_c)^{-\gamma} = D_0 c_c^{-\gamma}\phi_c^{-\gamma}n^{\frac{\gamma}{2}}$, where $c_c^{-\gamma}\phi_c^{-\gamma}n^{\frac{\gamma}{2}}$ can be interpreted as the probability that a consumer decides to shop. The problem faced by the retailer can then be stated as follows:

$$\max_n z_r(n) = (p - \tilde{c}_r\phi_r n^{\frac{1}{2}})D_0 c_c^{-\gamma}\phi_c^{-\gamma}n^{\frac{\gamma}{2}}, \tag{3.27}$$

where p denotes the unit selling price, and $\tilde{c}_r = c_r + t\hat{c}_r$. Treating n as a continuous variable, we can show that the optimal solution to (3.27) is given by:

$$n^* = [\frac{\gamma p}{(\gamma+1)\tilde{c}_r\phi_r}]^2.$$

We can see that n^* is decreasing in the emission penalty t. That is, the higher is the penalty, the fewer is the number of stores, as doing so reduces the retailer's transportation distances. However, by doing so, the retailer affects consumers in two ways. It reduces the fraction of consumers who shop and increases the traveling distances for those who do. Therefore, if the reduction in demand is

[4] For example, the cost and emission parameters could be determined as follows: $c_i = \frac{v_i + f_i p_i}{q_i}$ and $\hat{c}_i = \frac{f_i e_i}{q_i}$, where v_i is the non-fuel cost incurred by a transport vehicle per unit of distance, f_i is the amount of fuel consumed per unit distance traveled, p_i is the price per unit of fuel, e_i is the emissions per unit of fuel consumed, and q_i is the load carried per vehicle (see further discussion in Cachon (2014)).

sufficiently small (or the emission-intensity of consumer travel is sufficiently high), it may be possible for consumer-related emissions to increase. It may also be possible for this increase to be higher than the decrease in the retailer's emissions. In what follows, we provide a necessary and sufficient condition for this to occur.

Let e_{SC} denote total supply chain emission (the sum of retailer and consumer emissions). Then,

$$e_{SC} = (\hat{c}_r \phi_r n^{*\frac{1}{2}} + \hat{c}_c \phi_c n^{*-\frac{1}{2}}) D_0 c_c^{-\gamma} \phi_c^{-\gamma} n^{*\frac{\gamma}{2}}.$$

The following proposition directly follows from differentiating e_{SC} with respect to t.

Theorem 7 *Total supply chain emission e_{SC} increases in the emission penalty t if and only if*

$$\gamma \leq \sqrt{\frac{\tilde{c}_r^2 \hat{c}_c \phi_r \phi_c}{p^2 \hat{c}_r + \tilde{c}_r^2 \hat{c}_c \phi_r \phi_c}}.$$

The above result indicates that total supply chain emission would increase if consumers are sufficiently insensitive to traveling distance as measured by the parameter γ. The threshold than which gamma must be smaller is increasing in the emission-intensity of consumers as specified by $\hat{c}_c$.

Results similar to Proposition 7 can still be obtained when other features are added to the model, including a penalty on emissions borne by the consumers or fixed costs associated with operating each store for the retailer. A similar result can also be obtained for systems where the retailer decides on the selling price and consumers are sensitive to both cost of travel and price. Finally, results can be obtained, although not analytically, for the case where n is treated as a discrete variable. Details for these cases are again omitted for the sake of brevity.

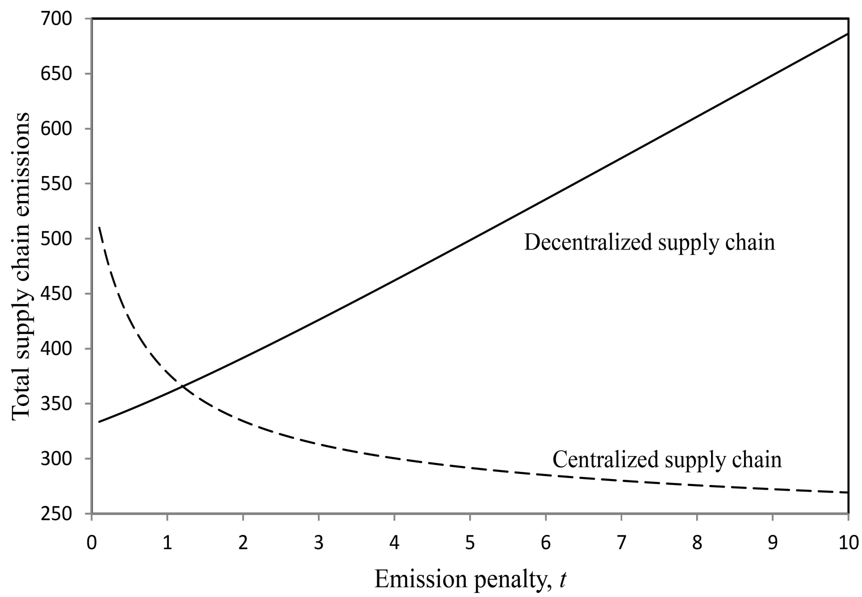


Figure 3.7: The impact of the emission penalty on supply chain emission for an example store density problem ($p = 4, D_0 = 1000, c_c = 0.5, c_r = 0.1, \hat{c}_c = 1, \hat{c}_r = 0.02, \phi_c = 0.7652, \phi_r = 1, \gamma = 0.1$).

The potential negative impact of emission penalties on supply chain emission could be mitigated if the retailer were to bear the cost of emissions for the entire supply chain (its own and those of the consumers). It would also be mitigated if consumers were to be penalized for their own emissions and those of the retailer (e.g., consumers pay an emission tax on both the fuel they purchase to travel and the emission associated with the products they buy from the retailer). The effect would of course also be mitigated if the retailer took on the full cost of the supply chain (its own and that of the consumers) in deciding on the number of stores. This is the case treated by Cachon (2014) who models a system where the retailer takes on the role of a central planner and compensates consumers for their full travel cost, including their emission cost if any. As we discussed in Section 4.2, a centralized supply chain may not however guarantee that total supply chain emission would be lower than that of a decentralized supply chain. Figure 3.7 illustrates this for an example system.

3.6.3 A General Model Formulation

In this section, we briefly describe a general model formulation of which the various models we have considered so far can be viewed as special cases. We do so to illustrate the broader applicability of the results we have obtained and to further explain how the phenomenon of higher emission penalties leading to higher supply chain emissions may arise.

Consider a supply chain consisting of two firms, which we denote by firm 1 and firm 2 respectively. (As mentioned earlier, we use the term firm loosely to refer to economic agents which may include independent firms, business units within a single firm, or consumers.) Firm i takes an action, an operational decision in our context, which we denote by x_i, within a feasible set S_i for $i = 1, 2$. The action taken by a firm, in addition to affecting its profit and emissions, affects the profit and emissions of the other firm and we let $f_i(x_1, x_2)$ and $e_i(x_1, x_2)$ denote respectively the profit and emissions of firm i for $i = 1, 2$.

This formulation of course allows for the possibility of the action of only one firm affecting the profit and emissions of the other. The buyer-supplier problem we studied falls in this category with only the buyer's action (the order quantity) affecting the supplier. On the other hand, the make versus buy and the store density problems fall in the general category. In the make versus buy problem, the amount to produce in-house decided by the manufacturer affects the profit and emissions of the supplier and, similarly, the wholesale price decided by the supplier affects the profit and emissions of the manufacturer. Likewise, in the store density problem, the number of stores chosen by the retailer affects the travel costs and emissions of the consumers, while the collective action by the consumers on whether or not to shop, modeled implicitly via the demand function, affects the profit and emissions of the retailer.

Each firm i ($i = 1, 2$) is subject to an emission penalty t_i per unit of emissions (note that this allows for $t_1 = t_2 = t$ or either t_1 or t_2 being zero, the cases treated in the paper). The firms engage in a Stackelberg game (the analysis can be extended to other forms of strategic interaction, such as when the firms choose their actions simultaneously). Firm 2 observes Firm 1's action x_1 and chooses x_2 to maximize its after penalty profit. Firm 1, anticipating the action of firm 2, chooses x_1 to maximize its own after penalty profit. If we consider settings where Firm 2's best response function and Firm 1's optimal solution, which we denote as $x_2^*(x_1)$ and x_1^*, are unique, as they are in the models we have considered, then the problem faced by firm 2, given the choice x_1 of firm 1, can be specified as follows:

$$\max_{x_2 \in S_2} z_2(x_1, x_2) = f_2(x_1, x_2) - t_2 e_2(x_1, x_2), \tag{3.28}$$

while the problem faced by firm 1 can be specified as:

$$\max_{x_1 \in S_1} z_1(x_1, x_2^*(x_1)) = f_1(x_1, x_2^*(x_1)) - t_1 e_1(x_1, x_2^*(x_1)). \tag{3.29}$$

In equilibrium, this leads to a total supply chain emission, which we denote by e_{SC}, given by

$$e_{SC}(x_1^*, x_2^*) = e_1(x_1^*, x_2^*) + e_2(x_1^*, x_2^*), \tag{3.30}$$

where (x_1^*, x_2^*) is the equilibrium solution.

From the above equation, we can see that whether higher emission penalties lead to higher or lower supply chain emission depends on how the individual actions of the firms in equilibrium affect the emissions of both firms. In the case where the supply chain emission function e_{SC} is continuous and differentiable in t_i, which is again the case in the models we studied, the necessary and sufficient condition for supply chain emission to increase with t_i is given by

$$\frac{de_{SC}(x_1^*, x_2^*)}{dt_i} = \frac{de_1(x_1^*, x_2^*)}{dt_i} + \frac{de_2(x_1^*, x_2^*)}{dt_i} \geq 0. \tag{3.31}$$

The above condition holds in two cases. The first is when the emissions of both firms increase as the emission penalty increases. The second is when the emissions of one firm decrease but those of the other firm increase, with the rate of increase for one firm higher than the rate of decrease for the other firm (this would be the case when the action of only one of the firms affects the emissions of the other firm, and not vice-versa).

It is easy to verify that the above condition is what leads to the conditions described in Theorems 3-7 for the models we studied. We can easily recast much of the discussion of the remedies considered in terms of this general formulation. We can also extend the model formulation to consider settings where each firms takes a vector of actions instead of just one (to accommodate, for example, situations where firms make both operational and pricing decisions, as in the model of Section 5 with endogenous demand). Given the generality of the model formulation, we expect that there would be various other applications that would conform to the specification of the model and exhibit similar supply chain effects.

3.7 Summary and Concluding Comments

In this paper, we examined the impact of emission penalties in decentralized supply chains. We showed that, perhaps surprisingly, emission penalties (if they are imposed on the emission of each firm) can lead to higher overall supply chain emission. We showed that this is because firms, in their efforts to reduce their own emissions, can make decisions that end up increasing the emissions of other firms. Although we based our analysis primarily on a buyer-supplier model where the decisions the firms make involve choosing order quantities, we showed that the same effects arise in other settings. We also described a general model formulation of which the various models we studied can be viewed as special cases. We described potential remedies that involve internalizing, to varying degrees, the emission externality a firm may impose on another firm, and analyzed the performance of these remedies in terms of emission, supply chain profit, and social welfare. Although all the remedies guarantee that higher penalties lead to lower emissions, we showed that they do not guarantee that the resulting emissions would be lower than those obtained when firms are penalized only for their own emissions.

A contribution of the paper is in shedding light on an effect that, to our knowledge, has not been widely studied. Accounting for this effect appears important in assessing whether or not well-meaning initiatives undertaken by individual firms or policies enacted by governments lead to their intended outcomes. Our results provide support for efforts to hold firms accountable not only for their own emissions but for the emissions of their suppliers and customers and for the development of standards for documenting such emissions.

Although in much of our analysis we have treated settings where the firms are motivated by an explicit penalty on emissions (or consumption), the effect we document is also present when firms or business units decide to reduce their emissions for other reasons (e.g., for reputational reasons). In this case again, a firm in its pursuit of emission reduction can create externalities that lead other firms to emit more and result in a net increase in total emission. Also, in much of our analysis, we have assumed that all firms in the supply chain are subject to a common price on emissions. The results we describe however hold even if only one firm in the supply chain is subject to a penalty or if the penalties are different for different firms.

In this paper, we considered settings where firms affect emissions by making either operational or pricing adjustments. Firms of course may also invest in technology that affects their underlying emission parameters for production, transportation, and storage, among others. Our analysis can be extended to those settings and to settings where firms first make technology investments and then operational adjustments. In fact, it is not difficult to find instances where such investments can amplify the negative impact of emission penalties (e.g., having affected a change in technology, a firm finds it less costly to make operational decisions that adversely affect the emissions of other firms).

Avenues for future research are many. It would be useful to investigate other applications where the effect of higher penalties leading to higher emissions is particularly prominent. It would also be useful to identify other remedies that are easy to implement and manage. In settings in practice where firms have undertaken emission reduction efforts, it would be of interest to empirically investigate whether their suppliers and customers saw increases or decreases in their emissions.

Chapter 4

Work-In-Progress and Future Research Directions

In this chapter, we describe work-in-progress and future research directions, consisting of two parts. In the first part, we consider a model where the price of carbon is stochastic. The second part deals with two models where demand is no longer assumed to be exogenous. For the sake of brevity, we omit all the technical developments and simply highlight some of the main results obtained so far.

4.1 Systems with Stochastic Carbon Prices

In this section, we describe preliminary analysis we carried out to understand better the impact of variability in carbon prices. In Chapter 3, we considered a setting where there is a tax on emission and this tax, which corresponds to the price of carbon, is fixed. However, there are settings where the price of carbon may vary over time. For example, this is the case under a *cap-and-trade system* where the emission of firms are capped but firms are allowed to buy and sell emission credits, based on whether they exceed or fall below their emission caps. The analysis we carry out is also more generally relevant to settings where the cost parameters (purchasing, fixed ordering, and holding costs) are stochastic. There is a rich literature dealing with purchase cost variability (see for example Ho et al. (1998) and Berling and Martínez-de-Albéniz (2011)). However, very little of this literature addresses settings where variability is also associated with fixed costs and inventory carrying costs. In this section, we use the framework of the economic order quantity (EOQ) model as discussed in Chapter 2 to investigate the impact of cost variability.

4.1.1 Preliminary Results

Consider the setting of Chapter 2. A price t is associated with emissions, except that t is now a random variable. Prices are assumed to be taking on discrete values $t_1, t_2, ..., t_S$ and are independently and identically distributed (i.i.d.) with π_i corresponding to the probability associated with price t_i, for $i = 1, 2, ..., N$.

In particular, we assume that the carbon price observed at any time is i.i.d. and can take on values from $\{t_1, t_2, ..., t_S\}$ with probabilities $\{\pi_1, \pi_2, ..., \pi_S\}$. We assume that at the start of each replenishment cycle, the current price is observed and then a decision on the order quantity is made. The objective of the firm is to choose an order quantity each time it observes price t_i such that its long run average cost per unit time is minimized.

Noting that the replenishment times form a renewal process, it can be shown that, for a given vector of order quantities $Q(t_1), Q(t_2), ..., Q(t_S)$, the long run average cost per unit time is given by

$$Z(Q(t_1), Q(t_2), ..., Q(t_S)) = \frac{1}{\sum_{s=1}^{S} \frac{Q(t_s)}{D}\pi_s}(A + \sum_{s=1}^{S}[(c + t_s\hat{c})Q(t_s) + t_s\hat{A} + \frac{(h + t_s\hat{h})Q^2(t_s)}{2D}]\pi_s), \tag{4.1}$$

where $Q(t_s)$ is the order quantity used if price t_s is used. The following theorem characterizes the optimal vector of order quantities.

Theorem 8 *The optimal ordering quantities* $\{Q^*(t_1), Q^*(t_2), ..., Q^*(t_S)\}$ *are given by*

$$Q^*(t_s) = \frac{1}{(h + t_s\hat{h})E[\frac{1}{h + t\hat{h}}]}(-\hat{c}DE[\frac{t_s - t}{h + t\hat{h}}] + \sqrt{\hat{c}^2D^2(E^2[\frac{t_s - t}{h + t\hat{h}}] - E[\frac{(t_s - t)^2}{h + t\hat{h}}]E[\frac{1}{h + t\hat{h}}]) + 2D(A + \hat{A}\mu_t)E[\frac{1}{h + t\hat{h}}]}), \tag{4.2}$$

and the optimal long run average cost per unit time is given

$$Z(Q^*(t_1), Q^*(t_2), ..., Q^*(t_S)) = \frac{D\hat{c}}{\hat{h}Y} + \frac{c\hat{h} - \hat{c}h}{\hat{h}} + \frac{D\sqrt{\frac{\hat{c}^2}{\hat{h}^2} - \frac{\hat{c}^2}{\hat{h}^2}(h + \hat{h}\mu_t)Y + \frac{2(A + \hat{A}\mu_t)Y}{D}}}{Y} \tag{4.3}$$

where $\mu_t = E[t]$ *is the expected carbon price, and* $Y = E[\frac{1}{h + t\hat{h}}]$.

Using Theorem 8, a number of results can be obtained, including the following:

- A system where the price of carbon is a random variable always leads to a lower expected cost than one where the price of carbon is fixed and equal to the expected value of the random price (a phenomenon due in part to the fact that price variability allows the firm to order less when the prices are low and more when prices are high). This results suggests that, everything else remaining the same, a system where the price of carbon is subject to variability (perhaps as in a cap-and-trade system) is more preferable to firms.

- Although a variable price is preferable to a fixed price, high variability (as measured by variance) does not always lead to lower cost. This is in contrast to the result that would be obtained if variability was only associated with the purchase cost. This effect is due to the fact that savings on purchasing less (more) when prices are higher (lower) must be traded-off against the corresponding savings in fixed ordering and holding costs.

- In general, higher variability is more preferable for settings where the expected cost function is concave in the carbon price. We can show this is true for several special cases, including the case when $\hat{h} = 0$ and when t is uniformly distributed. We can also show that higher variability is always preferable if it is measured using the stochastic convex ordering; more if $t \leq_{cv} \tilde{t}$, then $Z(t) \geq Z(\tilde{t})$.

4.1.2 Future Directions

We plan to extend the above analysis in several ways. First, we plan to consider systems where the firm does not necessarily incur the cost of carbon at the beginning of the replenishment period or as emissions are incurred. The firm may be accountable for its emissions at fixed points in time. We also plan to consider systems where firms are subject to a specified cap and can actively buy and sell emission credits, either continuously or at fixed points in time. Moreover, we note that variability in the price of emissions essentially reduces to variability in the cost parameters. In the analysis we carried out so far, we assume that the costs are perfectly correlated, we plan to study systems where this may not be the case, with different costs having varying degrees of correlation, either positive or negative.

4.2 Inventory Management with Emission Sensitive Demand

With their increasing awareness of environmental concerns and their negative impacts, consumers are beginning to exhibit preferences towards greener products, including products with less carbon footprints. Harris (2007) finds that “consumer demand for products that are clearly identified as genuinely sustainable, even though they may be perceived to be more expensive than traditional products”. Walmart’s success with its sustainability program is contributed partially to the additional revenue generated through a premium for its “green” products and the increased consumer demand (see for example Plambeck (2012)). Companies have implemented measures such as obtaining sustainability certifications (e.g., USDA Organic in the US), and labelling the carbon content of their products (e.g., Tesco). Such practices have allowed consumers to make more informed decisions which encourage them to purchase “green” products. Vanclay et al. (2011) for example confirm the positive impact of carbon labelling on the sales of grocery items.

As pointed out by Tang and Zhou (2012), studies on emissions and pollution reductions have largely ignored the market forces. As a result, there is a need for research addressing consumer response and preference towards greener products in the context of supply chain and operations management. Gosh and Shah (2012) study a two firm apparel supply chain facing consumers that prefer green products, and compare the green innovation and other decisions between various supply chain structures. There are a number of literature in the field of environmental economics incorporating consumers' prefernces towards green products. For example, Moraga-González and Padrón-Fumero (2002) study a duopoly facing consumers with varying preferences for green products. Conrad (2005) studies the effectiveness of environmental policies when considering consumer environmental awareness using a duopoly model.

In this section, we consider consumers' preference towards low emission products in the context of supply chain and inventory management.

4.2.1 Preliminary Results

We base the following analysis on the setup from Chapter 2 and consider the setting that customer demand is not only sensitive to the price of of the product but to the environmental image of the firm as well. Specifically, we assume that customer demand D is linearly decreasing in the price of the product p and the annual emission of the firm e: $D = d - ap - be$, with a and b being customers' level of sensitivity to price and emission respectively. The objective of the firm is to maximize the annual profit by choosing the order quantity Q:

$$\begin{aligned} &\max_{Q}\ \pi(Q) = (p-c)D - (\frac{AD}{Q} + \frac{hQ}{2}) \\ &s.t. \quad D = d - ap - be \end{aligned} \tag{4.4}$$

The optimal solution to (4.4) is given by:

$$Q^* = \sqrt{\frac{2(\frac{h\hat{A}}{1+b\hat{c}} + \frac{A}{p-c})(d - ap + \frac{b^2\hat{A}\hat{h}}{2(1+b\hat{c})})}{b\hat{h} + \frac{h(1+b\hat{c})}{p-c}}} - \frac{b\hat{A}}{1+b\hat{c}}.$$

Using the optimal solution, we obtained some preliminary results and summarize below:

- Moderate carbon emission sensitivity can have significant impact on regulating firms and reducing emissions, while high carbon sensitivity does not add much value in terms of emission reduction while it can significantly hurt firms profitability.
- In the case of $\hat{A} = 0$, we show that the optimal order quantity and corresponding emission decrease with the product price, while profit first increases and then decreases with the product price.
- In the case of $\hat{h} = 0$, we show that emission decreases with the product price, while the optimal order quantity first increases and then decreases with the product price.

4.2.2 Future Directions

There are several directions for future research. One is to study the effect of customer sensitivity on the emissions of the firm and extend the current results to more general settings. Secondly, it would be interesting to extend the analysis to incorporate interactions among firms within a supply chain and study how consumers' emission sensitivity affects supply chain decisions, performance as well as the social welfare. It would also be interesting to examine the effect of supply chain coordination on firms' profitability as well as emission abatement. Additionally, another useful avenue of research is to use advanced simulation tool to incorporate more fully the preferences of various parties as well as their interactions, through which more precise estimates of the implications of different environmental policies and supply chain inputs can be obtained.

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